

Chiral perturbation theory and lattice *a review ?*

Sébastien Descotes-Genon

Laboratoire de Physique Théorique
CNRS & Université Paris-Sud 11, Orsay (France)

June 7 2005



Contents

Lattice and χ PT

The chiral structures of QCD vacuum

Three-flavour chiral extrapolations

Relevance of lattice simulations

hep-ph/0410233

In collaboration with
L.Girlanda, N.Fuchs, J.Stern





What's up, doc ?

χ PT

Chiral Perturbation Theory { effective field theory of QCD
perturbation around $m_q = p = 0$

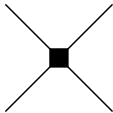
- ▶ left and right quark chiralities (\equiv helicities) decouple
- ▶ chiral symmetry $SU_L(N_f) \otimes SU_R(N_f)$ spontaneously broken
 $\rightarrow \pi, K, \eta$ Goldstone bosons
- ▶ observables expanded in powers of m_q and p

χ PT

Chiral Perturbation Theory { effective field theory of QCD
perturbation around $m_q = p = 0$

- ▶ left and right quark chiralities (\equiv helicities) decouple
- ▶ chiral symmetry $SU_L(N_f) \otimes SU_R(N_f)$ spontaneously broken
 $\rightarrow \pi, K, \eta$ Goldstone bosons
- ▶ observables expanded in powers of m_q and p

Chiral symmetry yields the structure of the interactions
but not the values of the couplings



$\pi\pi$ -scattering amplitude vanish for

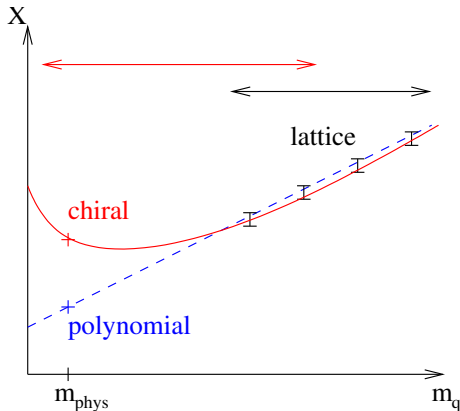
$$m_q, p \rightarrow 0$$
$$A_{\pi\pi} = a \cdot s + b \cdot M_\pi^2 + O(p^4)$$

but value of a and b ?

Lattice and χ PT

Overlap with lattice ?

- ▶ Light masses for χ PT, but unknown constants
- ▶ Heavier masses for lattice, but extrapolation



In particular, strong impact of *chiral logarithms* at NLO

$$4M_\pi^2 \log \frac{M_\pi^2}{\mu^2}$$

not seen in lattice simulations

Recent progress (1)

π, K, η only

- ▶ Two-loop computations for almost all quantities in full QCD, some in partially-quenched QCD
- ▶ Problem with NNLO low-energy constants (more than 100 !)
Model-dependence of the results ?
- ▶ Low-energy constants for electromagnetic and weak radiative processes using resonance saturation

πN

- ▶ Better control of analytic structure of results
(new regularisation schemes)
- ▶ Investigation of chiral extrapolation from lattice to real world
(mass of the nucleon, form factors)

Recent progress (2)

NN and few-nucleon systems

- ▶ chiral potential inside a Lippmann-Schwinger equation
 - bound states out of weak coupling description
 - cut-off to separate low and high energies
- ▶ Relation with *NN* potential and bound states on the lattice
 - discussions about the “best” implementation of χ PT

Finite-volume effects

- ▶ Lellouch-Lüscher formula and generalisations

$$M_X^2(L) - M_X^2(\infty) = \int d\nu K(\nu) T_{X\pi \rightarrow X\pi}^{\text{forward}}(\nu) + O(\exp(-L))$$

- ▶ ϵ regime: large pion in a small box $1/M_\pi \gg L \gg 1/F_\pi$

What equals the beauty of well-kept hair?



Is your scalp oily?

Shampoo the hair once each week, using the following procedure. Before you wet your hair with Wildroot Shampoo, run the scalp with your finger tips. Apply warm water, and let the steam, arrange lotion about the only day. Rinse thoroughly. When dry, massage the scalp with Wildroot Quinine Hair Dress.



Have you found dandruff?

Use at once since a wash for every even, every day with Wildroot Hair Tonic on the scalp. This should be done in the most thorough manner, putting the hair so as to reach every spot on the scalp and arranging gently with the fingers. Finally by drawing the hair with the comb, one strand at a time.

Is your scalp dry?

Once every other week, give yourself a treatment. Remove dandruff from scalp by applying Wildroot Hair Tonic. Then gently massage Wildroot Tarolium into the scalp. Before you use your hair. Cover your head with a towel for four minutes. With more Tarolium and warm water, shampoo the hair. Rinse well, and follow with cold water.



WILDROOT
HAIR PREPARATIONS

Advertisement

Three chiral limits of interest

↑ b,t

c

c

c

$m_u, m_d \rightarrow 0$

hadronic

scale

Λ_H

s

s

Λ_{QCD}

u,d

$N_f = 2^x$

d

u

$N_f = 3$

u,d,s

$N_f = 3$: $m_s \rightarrow 0$

$N_f = 2^x$: m_s physical

$N_f = 2^{\text{lat}}$: no dynamical s

Three chiral limits of interest

↑ b,t

c

c

c

$m_u, m_d \rightarrow 0$

hadronic

scale

Λ_H

s

s

Λ_{QCD}

$N_f = 3$: $m_s \rightarrow 0$

$N_f = 2^\chi$: m_s physical

$N_f = 2^{\text{lat}}$: no dynamical s

u,d

$N_f = 2^\chi$

d

u

$N_f = 3$

u,d,s

Two versions
of χ PT

$N_f = 2^\chi$: π only (few param. & processes)

$N_f = 3$: π, K, η (more param. & processes)

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \rightarrow 0} -\langle 0 | \bar{u}u | 0 \rangle \quad \left\{ \begin{array}{l} \Sigma(3) = \Sigma(2; 0) \\ \Sigma(2^x) = \Sigma(2; m_s^{\text{phys}}) \\ \Sigma(2^{\text{lat}}) = \Sigma(2; \infty) \end{array} \right.$$

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \rightarrow 0} -\langle 0 | \bar{u}u | 0 \rangle \quad \left\{ \begin{array}{l} \Sigma(3) = \Sigma(2; 0) \\ \Sigma(2^x) = \Sigma(2; m_s^{\text{phys}}) \\ \Sigma(2^{\text{lat}}) = \Sigma(2; \infty) \end{array} \right.$$

$$\Sigma(2; m_s) = \Sigma(2; 0) + m_s \frac{\partial \Sigma(2; m_s)}{\partial m_s} + O(m_s^2)$$

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \rightarrow 0} -\langle 0 | \bar{u}u | 0 \rangle \quad \left\{ \begin{array}{l} \Sigma(3) = \Sigma(2; 0) \\ \Sigma(2^X) = \Sigma(2; m_s^{\text{phys}}) \\ \Sigma(2^{\text{lat}}) = \Sigma(2; \infty) \end{array} \right.$$

$$\Sigma(2^X) = \Sigma(3) + m_s^{\text{phys}} \lim_{m_u, m_d \rightarrow 0} i \int d^4x \langle 0 | \bar{u}u(x) \bar{s}s(0) | 0 \rangle + O(m_s^2)$$

From 2 to 3 massless flavours

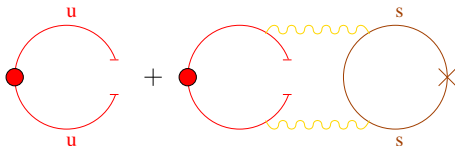
$$\Sigma(2; m_s) = \lim_{m_u, m_d \rightarrow 0} -\langle 0 | \bar{u}u | 0 \rangle \quad \left\{ \begin{array}{l} \Sigma(3) = \Sigma(2; 0) \\ \Sigma(2^x) = \Sigma(2; m_s^{\text{phys}}) \\ \Sigma(2^{\text{lat}}) = \Sigma(2; \infty) \end{array} \right.$$

$$\Sigma(2^x) = \Sigma(3) + m_s^{\text{phys}} \lim_{m_u, m_d \rightarrow 0} i \int d^4x \langle 0 | \bar{u}u(x) \bar{s}s(0) | 0 \rangle + O(m_s^2)$$

$\Sigma(2^x)$ contains

- ▶ A “genuine” condensate $\Sigma(3)$
- ▶ An “induced” condensate $m_s \times$ (scalar N_c -suppressed) effect from sea $s\bar{s}$ -pairs

SDG, L. Girlanda, J. Stern



From $K_{\ell 4}$, i.e., $\pi\pi$ scattering data

$$\frac{(m_u + m_d)\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08 \quad \dots \text{or larger}$$

SDG, L. Girlanda, N. Fuchs, J. Stern
G. Colangelo, J. Gasser, H. Leutwyler

From $K_{\ell 4}$, i.e., $\pi\pi$ scattering data

$$\frac{(m_u + m_d)\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08 \quad \text{SDG, L. Girlanda, N. Fuchs, J. Stern}$$

... or larger G. Colangelo, J. Gasser, H. Leutwyler

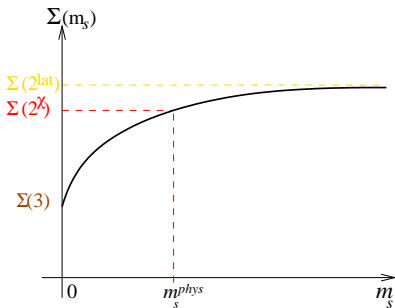
$$\underbrace{\langle \bar{u}u \rangle |_{m_{u,d}=0, m_s \text{ phys}}}_{\text{sizeable } \Sigma(2^\chi)} = \underbrace{\langle \bar{u}u \rangle |_{m_{u,d,s}=0}}_{\Sigma(3)} + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + \dots$$

From $K_{\ell 4}$, i.e., $\pi\pi$ scattering data

$$\frac{(m_u + m_d)\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08 \quad \text{SDG, L. Girlanda, N. Fuchs, J. Stern}$$

... or larger G. Colangelo, J. Gasser, H. Leutwyler

$$\underbrace{\langle \bar{u}u \rangle}_{\text{sizeable } \Sigma(2^\chi)} \Big|_{m_{u,d}=0, m_s \text{ phys}} = \underbrace{\langle \bar{u}u \rangle}_{\Sigma(3)} \Big|_{m_{u,d,s}=0} + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + \dots$$

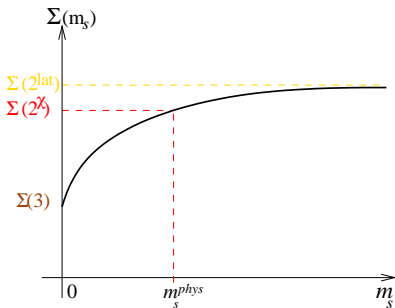


From $K_{\ell 4}$, i.e., $\pi\pi$ scattering data

$$\frac{(m_u + m_d)\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08 \quad \dots \text{ or larger}$$

SDG, L. Girlanda, N. Fuchs, J. Stern
G. Colangelo, J. Gasser, H. Leutwyler

$$\underbrace{\langle \bar{u}u \rangle}_{\text{sizeable } \Sigma(2^\chi)} \Big|_{m_{u,d}=0, m_s \text{ phys}} = \underbrace{\langle \bar{u}u \rangle}_{\Sigma(3)} \Big|_{m_{u,d,s}=0} + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + \dots$$



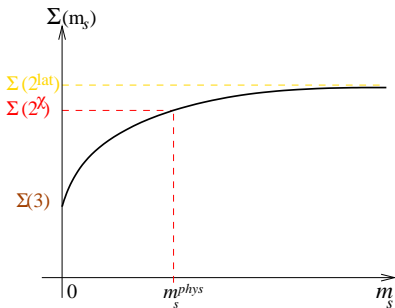
$\Sigma(3) \simeq \Sigma(2^\chi)$ and $\langle (\bar{u}u)(\bar{s}s) \rangle$ small
Zweig rule OK for scalars
No impact of strange sea quarks

From $K_{\ell 4}$, i.e., $\pi\pi$ scattering data

$$\frac{(m_u + m_d)\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08 \dots \text{or larger}$$

SDG, L. Girlanda, N. Fuchs, J. Stern
G. Colangelo, J. Gasser, H. Leutwyler

$$\underbrace{\langle \bar{u}u \rangle}_{\text{sizeable } \Sigma(2^\chi)}|_{m_{u,d}=0, m_s \text{ phys}} = \underbrace{\langle \bar{u}u \rangle}_{\Sigma(3)}|_{m_{u,d,s}=0} + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + \dots$$



$\Sigma(3) \simeq \Sigma(2^\chi)$ and $\langle (\bar{u}u)(\bar{s}s) \rangle$ small
Zweig rule OK for scalars
No impact of strange sea quarks

or

$\Sigma(3) < \Sigma(2^\chi)$ and $\langle (\bar{u}u)(\bar{s}s) \rangle$ large
Large Zweig-rule violation
Strange sea quarks important

Consequences for three-flavour chiral series

$$F_\pi^2 M_\pi^2 = 2m\Sigma(3) + 64m(m_s + 2m)B_0^2 \Delta L_6 + 64m^2 B_0^2 \Delta L_8 + O(m_q^2)$$

- ▶ $B_0 = -\lim_{m_u, m_d, m_s \rightarrow 0} \langle \bar{u}u \rangle / F_\pi^2 \quad m = m_u = m_d$
- ▶ $\Delta L_8 = L_8(M_\rho) + 0.20 \cdot 10^{-3} = O(p^4) \text{ LEC} + \chi \log$
- ▶ $\Delta L_6 = L_6(M_\rho) + 0.26 \cdot 10^{-3} = O(p^4) \text{ LEC} + \chi \log$

Consequences for three-flavour chiral series

$$F_\pi^2 M_\pi^2 = 2m\Sigma(3) + 64m(m_s + 2m)B_0^2 \Delta L_6 + 64m^2 B_0^2 \Delta L_8 + O(m_q^2)$$

- ▶ $B_0 = -\lim_{m_u, m_d, m_s \rightarrow 0} \langle \bar{u}u \rangle / F_\pi^2 \quad m = m_u = m_d$
- ▶ $\Delta L_8 = L_8(M_\rho) + 0.20 \cdot 10^{-3} = O(p^4) \text{ LEC} + \chi \log$
- ▶ $\Delta L_6 = L_6(M_\rho) + 0.26 \cdot 10^{-3} = O(p^4) \text{ LEC} + \chi \log$

L_6 is the awesome guy here

- ▶ Enhanced by m_s , related to $\langle (\bar{u}u)(\bar{s}s) \rangle \dots$
- ▶ ...and “guestimated” **assuming** Zweig rule in scalar sector

Possible numerical competition between $O(p^2)$ and $O(p^4)$
 $2mB_0 = M_\pi^2 + \dots$ may not be a good approximation

No decisive evidence to choose between the scenarios

- ▶ In the scalar sector, Zweig rule and large- N_c badly violated
- ▶ Large dispersive estimates of $\langle(\bar{u}u)(\bar{s}s)\rangle$
- ▶ πK scattering slightly favours significant role of sea $s\bar{s}$ pairs

B.Moussallam;SDG;P.Büttiker

No decisive evidence to choose between the scenarios

- ▶ In the scalar sector, Zweig rule and large- N_c badly violated
- ▶ Large dispersive estimates of $\langle(\bar{u}u)(\bar{s}s)\rangle$
- ▶ πK scattering slightly favours significant role of sea $s\bar{s}$ pairs

B.Moussallam;SDG;P.Büttiker

Large effect of $s\bar{s}$ pairs \implies difficult convergence of $SU(3)$ series

No decisive evidence to choose between the scenarios

- ▶ In the scalar sector, Zweig rule and large- N_c badly violated
- ▶ Large dispersive estimates of $\langle(\bar{u}u)(\bar{s}s)\rangle$
- ▶ πK scattering slightly favours significant role of sea $s\bar{s}$ pairs

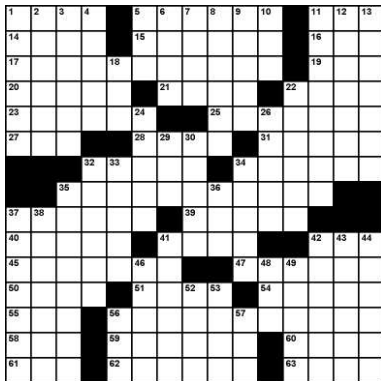
B.Moussallam;SDG;P.Büttiker

Large effect of $s\bar{s}$ pairs \implies difficult convergence of $SU(3)$ series

- ▶ Assume overall convergence for a subset of observables
- ▶ Leave open a numerical competition between LO and NLO
- ▶ Use only chiral couplings of the Lagrangian (F_0, B_0, L_i)
- ▶ Reexpress them in terms of $M_\pi^2, F_\pi^2 \dots$ only if **physical** motivation (nonanalytic poles, cuts, unitarity...)

$$M_\pi^2 \neq 2mB_0 !$$

SDG,L.Girlanda,N.Fuchs,J.Stern



Crossword corner

Three-flavour unquenched lattice

Idea: Unquenched lattice
to probe m_s -enhanced Zweig-rule violating effects

Three-flavour unquenched lattice

Idea: Unquenched lattice
to probe m_s -enhanced Zweig-rule violating effects

	Real QCD	Lattice	
2+1 flavours	(m, m, m_s)	$(\tilde{m}, \tilde{m}, m_s)$	$m \leq \tilde{m} \leq m_s$
Observables	X	\tilde{X}	$F_\pi^2, F_K^2, F_\pi^2 M_\pi^2, F_K^2 M_K^2$

Three-flavour unquenched lattice

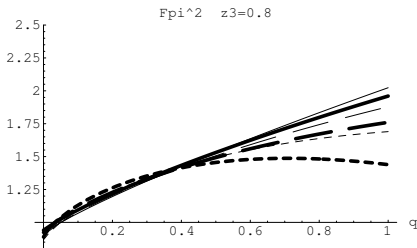
Idea: Unquenched lattice
to probe m_s -enhanced Zweig-rule violating effects

	Real QCD	Lattice	
2+1 flavours	(m, m, m_s)	$(\tilde{m}, \tilde{m}, m_s)$	$m \leq \tilde{m} \leq m_s$
Observables	X	\tilde{X}	$F_\pi^2, F_K^2, F_\pi^2 M_\pi^2, F_K^2 M_K^2$

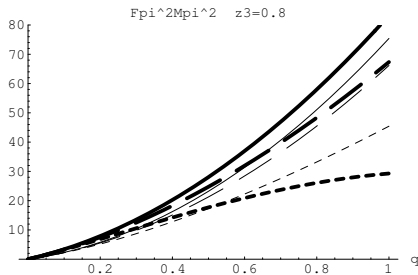
Real QCD: from chiral series, up to (small) NNLO remainders

$$O(p^4) \text{ LECs } L_{4,5,6,8} = \mathcal{F} \left[X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}, Z(3) = \frac{F_0^2}{F_\pi^2}, r = \frac{m_s}{m} \right]$$

Lattice: additional parameter: $q = \frac{\tilde{m}}{m_s} \sim "1/\tilde{r}"$



$$\tilde{F}_{\pi}^2 = \mathcal{F}(q = \tilde{m}/m_s)$$



$$\tilde{F}_{\pi}^2 \tilde{M}_{\pi}^2 = \mathcal{F}(q = \tilde{m}/m_s)$$

- ▶ Infinite volume, continuum limit, no NNLO remainders
- ▶ Zweig-rule violation in 0^{++} :
from none (full) to almost maximal (dotted)
- ▶ Varying $r = m_s/m$: 20 (thin) and 30 (thick)

... and similar results for kaons

Ratios of interest

What about finite-volume corrections ? LO computed within χ PT

For $L \sim 2.5$ fm, and any Zweig-rule violation due to $s\bar{s}$,

finite-volume corrections to $F_\pi^2 M_\pi^2$ and $F_K^2 M_K^2 < 10\%$

Ratios of interest

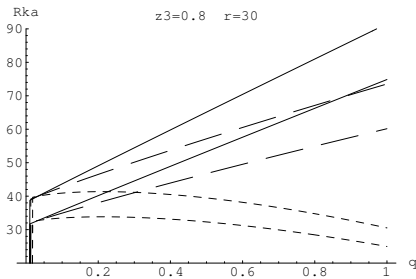
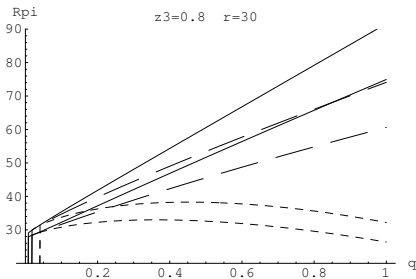
What about finite-volume corrections ? LO computed within χ PT

For $L \sim 2.5$ fm, and any Zweig-rule violation due to $s\bar{s}$,

finite-volume corrections to $F_\pi^2 M_\pi^2$ and $F_K^2 M_K^2 < 10\%$

$$R_\pi = \frac{1}{q} \frac{\tilde{F}_\pi^2 \tilde{M}_\pi^2}{F_\pi^2 M_\pi^2}$$

$$R_K = \frac{2}{(q+1)} \frac{\tilde{F}_K^2 \tilde{M}_K^2}{F_K^2 M_K^2}$$



to assess sea $s\bar{s}$ pairs, negligible (full) or significant (dashed) ?



The end of a review ?

Conclusions

Conclusions

- ▶ Two chiral limits
 $N_f = 3 : m_u, m_d, m_s \rightarrow 0$
 $N_f = 2^X : m_u, m_d \rightarrow 0, m_s \text{ physical}$

$$\Sigma(2^X) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

Conclusions

- ▶ Two chiral limits $N_f = 3 : m_u, m_d, m_s \rightarrow 0$
 $N_f = 2^X : m_u, m_d \rightarrow 0, m_s \text{ physical}$

$$\Sigma(2^X) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

- ▶ Role of sea $s\bar{s}$ -pairs $\leftrightarrow N_f$ -dependence of order parameters
 \leftrightarrow Zweig rule violation in scalar sector

Conclusions

- ▶ Two chiral limits $N_f = 3 : m_u, m_d, m_s \rightarrow 0$
 $N_f = 2^X : m_u, m_d \rightarrow 0, m_s \text{ physical}$

$$\Sigma(2^X) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

- ▶ Role of sea $s\bar{s}$ -pairs $\leftrightarrow N_f$ -dependence of order parameters
 \leftrightarrow Zweig rule violation in scalar sector
- ▶ Numerical competition between LO and NLO in chiral series
 \implies Care required to deal with 3-flavour chiral expansion

Conclusions

- ▶ Two chiral limits $N_f = 3 : m_u, m_d, m_s \rightarrow 0$
 $N_f = 2^x : m_u, m_d \rightarrow 0, m_s \text{ physical}$

$$\Sigma(2^x) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

- ▶ Role of sea $s\bar{s}$ -pairs $\leftrightarrow N_f$ -dependence of order parameters
 \leftrightarrow Zweig rule violation in scalar sector
- ▶ Numerical competition between LO and NLO in chiral series
 \implies Care required to deal with 3-flavour chiral expansion
- ▶ No direct experimental information on size of the effect (yet!)
 \implies Lattice simulations with three dynamical flavours
e.g., dependence of hadron observables on quark masses

Conclusions

- ▶ Two chiral limits
 $N_f = 3 : m_u, m_d, m_s \rightarrow 0$
 $N_f = 2^X : m_u, m_d \rightarrow 0, m_s \text{ physical}$

$$\Sigma(2^X) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

- ▶ Role of sea $s\bar{s}$ -pairs $\leftrightarrow N_f$ -dependence of order parameters
 \leftrightarrow Zweig rule violation in scalar sector
- ▶ Numerical competition between LO and NLO in chiral series
 \implies Care required to deal with 3-flavour chiral expansion
- ▶ No direct experimental information on size of the effect (yet!)
 \implies Lattice simulations with three dynamical flavours
e.g., dependence of hadron observables on quark masses
- ▶ Finite-volume effects controlled for sufficiently large volumes

Conclusions

- ▶ Two chiral limits $N_f = 3 : m_u, m_d, m_s \rightarrow 0$
 $N_f = 2^x : m_u, m_d \rightarrow 0, m_s \text{ physical}$

$$\Sigma(2^x) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

- ▶ Role of sea $s\bar{s}$ -pairs $\leftrightarrow N_f$ -dependence of order parameters
 \leftrightarrow Zweig rule violation in scalar sector
- ▶ Numerical competition between LO and NLO in chiral series
 \implies Care required to deal with 3-flavour chiral expansion
- ▶ No direct experimental information on size of the effect (yet!)
 \implies Lattice simulations with three dynamical flavours
e.g., dependence of hadron observables on quark masses
- ▶ Finite-volume effects controlled for sufficiently large volumes

Be careful with lattice extrapolations to light masses

AUTRANS 2005

GdR Physique subatomique
& calculs sur réseau

6 et 7 Juin 2005 Vercors

Calculs sur réseau &

- > Paramètres de QCD
- > Symétrie chirale
- > Physique des saveurs
- > Spectroscopie
- > Structure des baryons
- > Collisions d'ions lourds
- > Physique nucléaire

Contacts

- > Jaume Carbonell
carbonel@lpsc.in2p3.fr
- > Laurent Lellouch
lellouch@cpt.univ-mrs.fr
- > Madeleine Soyeur
msoyeur@cea.fr
- > <http://gdr-lqcd.in2p3.fr>

Avec le soutien de
CNRS-SPM, IN2P3, CEA

More soon
on your screens !

AUTRANS 2005

GdR Physique subatomique
& calculs sur réseau

6 et 7 Juin 2005 Vercors

Calculs sur réseau &

- > Paramètres de QCD
- > Symétrie chirale
- > Physique des saveurs
- > Spectroscopie
- > Structure des baryons
- > Collisions d'ions lourds
- > Physique nucléaire

Contacts

- > Jaume Carbonell
carbonel@lpsc.in2p3.fr
- > Laurent Lellouch
lellouch@cpt.univ-mrs.fr
- > Madeleine Soyeur
msoyeur@cea.fr
- > <http://gdr-lqcd.in2p3.fr>

Avec le soutien de
CNRS-SPM, IN2P3, CEA

More soon
on your screens !

Autrans 2006 ?