Chiral perturbation theory and lattice

a review ?

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In collaboration with
L.Girlanda, N.Fuchs, J.Stern

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Two Airlines Push for Expansion

What's up, doc ?

χPT

χiral Perturbation Theory $\begin{cases} \text{effective field theory of QCD} \\ \text{perturbation around } m_q = p = 0 \end{cases}$

- left and right quark chiralities ($\equiv$ helicities) decouple
- chiral symmetry $SU_L(N_f) \otimes SU_R(N_f)$ spontaneously broken
  $\rightarrow \pi, K, \eta$ Goldstone bosons
- observables expanded in powers of m_q and p

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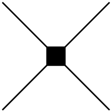
Chiral symmetry yields the structure of the interactions
but not the values of the couplings



$\pi\pi$-scattering amplitude vanish for
$m_q, p \rightarrow 0$

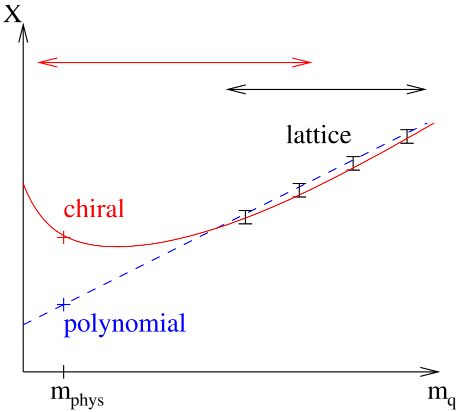
$$A_{\pi\pi} = a \cdot s + b \cdot M_\pi^2 + O(p^4)$$

but value of a and b ?

Lattice and χPT

Overlap with lattice ?

- Light masses for χPT, but unknown constants
- Heavier masses for lattice, but extrapolation



In particular, strong impact of *chiral logarithms* at NLO

$$4M_\pi^2 \log \frac{M_\pi^2}{\mu^2}$$

not seen in lattice simulations

Recent progress (1)

π, K, η only

- Two-loop computations for almost all quantities in full QCD, some in partially-quenched QCD
- Problem with NNLO low-energy constants (more than 100 !)
  Model-dependence of the results ?
- Low-energy constants for electromagnetic and weak radiative processes using resonance saturation

πN

- Better control of analytic structure of results (new regularisation schemes)
- Investigation of chiral extrapolation from lattice to real world (mass of the nucleon, form factors)

Recent progress (2)

NN and few-nucleon systems

- chiral potential inside a Lippmann-Schwinger equation
  - $\rightarrow$ bound states out of weak coupling description
  - $\rightarrow$ cut-off to separate low and high energies
- Relation with *NN* potential and bound states on the lattice
  - $\rightarrow$ discussions about the "best" implementation of χPT

Finite-volume effects

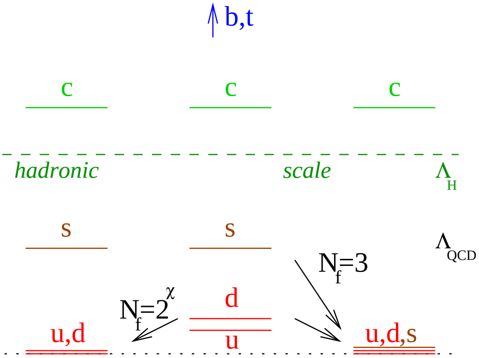
- Lellouch-Lüscher formula and generalisations

$$M_X^2(L) - M_X^2(\infty) = \int d\nu K(\nu)\, T^{\mathrm{forward}}_{X\pi\to X\pi}(\nu) + O(\exp(-L))$$

- ϵ regime: large pion in a small box $1/M_\pi \gg L \gg 1/F_\pi$

Advertisement

Three chiral limits of interest



$m_u, m_d \longrightarrow 0$

$N_f = 3$: $m_s \longrightarrow 0$

$N_f = 2^{\chi}$: m_s physical

$N_f = 2^{\mathrm{lat}}$: no dynamical s

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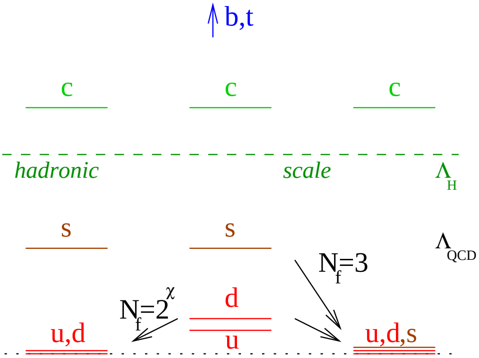

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Two versions of χPT

$N_f = 2^\chi$: π only (few param. & processes)

$N_f = 3$: π, K, η (more param. & processes)

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \to 0} -\langle 0|\bar{u}u|0\rangle \qquad \left\{ \begin{array}{rcl} \Sigma(3) & = & \Sigma(2; 0) \\ \Sigma(2^\chi) & = & \Sigma(2; m_s^{\mathrm{phys}}) \\ \Sigma(2^{\mathrm{lat}}) & = & \Sigma(2; \infty) \end{array} \right.$$

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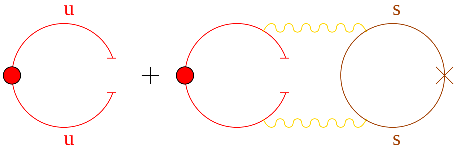
SDG,L.Girlanda,J.Stern

$\Sigma(2^\chi)$ contains

- A "genuine" condensate $\Sigma(3)$
- An "induced" condensate $m_s \times$ (scalar N_c-suppressed) *effect from sea $s\bar{s}$-pairs*



From $K_{\ell 4}$, i.e., $\pi\pi$ scattering data

$$\frac{(m_u + m_d)\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08$$ SDG,L.Girlanda,N.Fuchs,J.Stern

... or larger G.Colangelo,J.Gasser,H.Leutwyler

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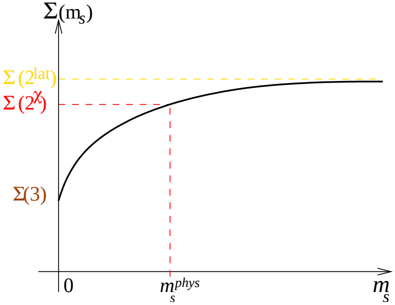

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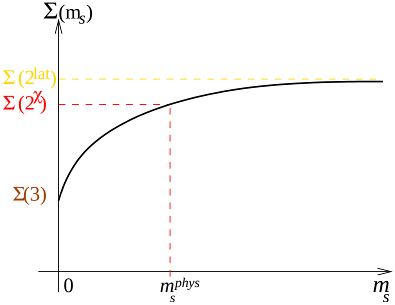

$\Sigma(3) \simeq \Sigma(2^\chi)$ and $\langle (\bar{u}u)(\bar{s}s) \rangle$ small

Zweig rule OK for scalars

No impact of strange sea quarks

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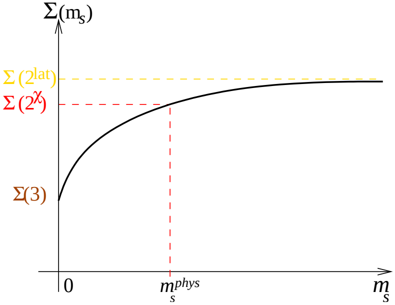

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or

$\Sigma(3) < \Sigma(2^\chi)$ and $\langle(\bar{u}u)(\bar{s}s)\rangle$ large

Large Zweig-rule violation

Strange sea quarks important

Consequences for three-flavour chiral series

$$F_\pi^2 M_\pi^2 = 2m\Sigma(3) + 64m(m_s+2m)B_0^2\Delta L_6 + 64m^2 B_0^2 \Delta L_8 + O(m_q^2)$$

- $B_0 = -\lim_{m_u,m_d,m_s\to 0}\langle \bar{u}u\rangle/F_\pi^2 \qquad m = m_u = m_d$
- $\Delta L_8 = L_8(M_\rho) + 0.20\cdot 10^{-3} = O(p^4)\ \mathrm{LEC} + \chi\log$
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L_6 is the awesome guy here

- Enhanced by m_s, related to $\langle(\bar{u}u)(\bar{s}s)\rangle\ldots$
- ... and "guestimated" assuming Zweig rule in scalar sector

Possible numerical competition between $O(p^2)$ and $O(p^4)$
$2mB_0 = M_\pi^2 + \ldots$ may not be a good approximation

No decisive evidence to choose between the scenarios

- In the scalar sector, Zweig rule and large-N_c badly violated
- Large dispersive estimates of $\langle(\bar{u}u)(\bar{s}s)\rangle$
- πK scattering slightly favours significant role of sea $s\bar{s}$ pairs

B.Moussallam;SDG;P.Büttiker

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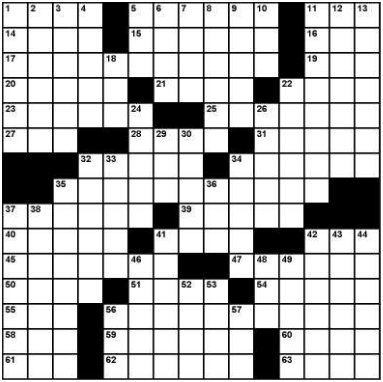
B.Moussallam;SDG;P.Büttiker

Large effect of $s\bar{s}$ pairs $\Longrightarrow$ difficult convergence of $SU(3)$ series

- Assume overall convergence for a subset of observables
- Leave open a numerical competition between LO and NLO
- Use only chiral couplings of the Lagrangian (F_0, B_0, L_i)
- Reexpress them in terms of $M_\pi^2, F_\pi^2 \ldots$ only if physical motivation (nonanalytic poles, cuts, unitarity. . .)

$M_\pi^2 \neq 2mB_0$!

SDG,L.Girlanda,N.Fuchs,J.Stern



Crossword corner

Three-flavour unquenched lattice

Idea: Unquenched lattice
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	Real QCD	Lattice	
2+1 flavours	(m, m, m_s)	$(\tilde{m}, \tilde{m}, m_s)$	$m \leq \tilde{m} \leq m_s$
Observables	X	$\tilde{X}$	$F_\pi^2, F_K^2, F_\pi^2 M_\pi^2, F_K^2 M_K^2$

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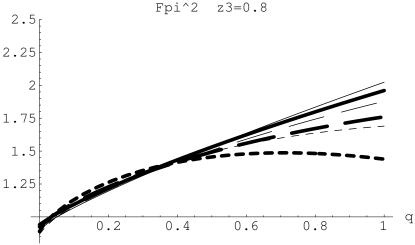
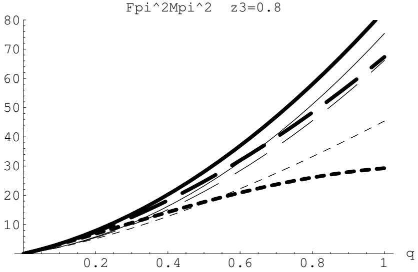
Real QCD: from chiral series, up to (small) NNLO remainders

$$O(p^4)\ \mathrm{LECs}\ L_{4,5,6,8} = \mathcal{F}\left[X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}, Z(3) = \frac{F_0^2}{F_\pi^2}, r = \frac{m_s}{m}\right]$$

Lattice: additional parameter: $q = \dfrac{\tilde{m}}{m_s} \sim$ "$1/\tilde{r}$"



$$\tilde{F}_\pi^2 = \mathcal{F}(q = \tilde{m}/m_s)$$



$$\tilde{F}_\pi^2 \tilde{M}_\pi^2 = \mathcal{F}(q = \tilde{m}/m_s)$$

- Infinite volume, continuum limit, no NNLO remainders
- Zweig-rule violation in 0^{++}:
  from none (full) to almost maximal (dotted)
- Varying $r = m_s/m$: 20 (thin) and 30 (thick)

. . . and similar results for kaons

Ratios of interest

What about finite-volume corrections ? LO computed within χPT
For $L \sim 2.5$ fm, and any Zweig-rule violation due to $s\bar{s}$,
finite-volume corrections to $F_\pi^2 M_\pi^2$ and $F_K^2 M_K^2 < 10\%$

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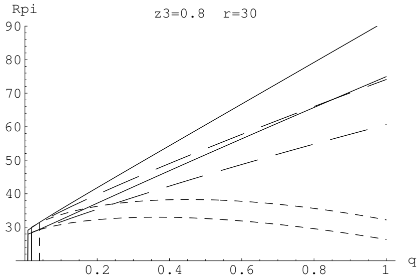
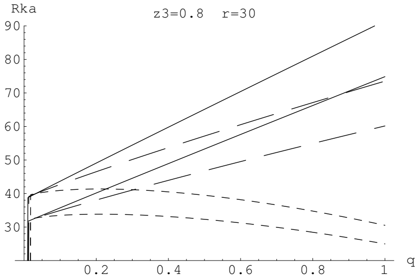
$$R_\pi = \frac{1}{q} \frac{\tilde{F}_\pi^2 \tilde{M}_\pi^2}{F_\pi^2 M_\pi^2} \qquad R_K = \frac{2}{(q+1)} \frac{\tilde{F}_K^2 \tilde{M}_K^2}{F_K^2 M_K^2}$$





to assess sea $s\bar{s}$ pairs, negligible (full) or significant (dashed) ?

The end of a review ?

Conclusions

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- Two chiral limits $\begin{array}{l} N_f = 3 : m_u, m_d, m_s \to 0 \\ N_f = 2^\chi : m_u, m_d \to 0, m_s \text{ physical} \end{array}$

$$\Sigma(2^\chi) = \Sigma(3) + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + O(m_s^2)$$

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Be careful with lattice extrapolations to light masses

More soon
on your screens !

More soon
on your screens !

Autrans 2006 ?