Physique des saveurs sur réseau

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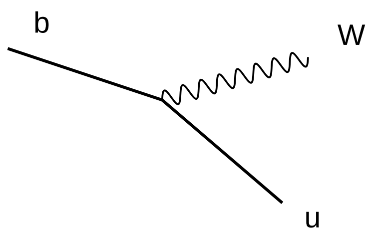
Motivation

Test SM paradigm of quark flavor mixing and CP violation and look for new physics

Unitary CKM matrix



$$\sim V_{ub} \rightarrow V = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 1-\frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array} + \mathcal{O}(\lambda^4)$$

$\lambda = \sin\theta_c \simeq 0.227(3)$ $\quad A \simeq 0.80(3)$ $\quad \sqrt{\rho^2+\eta^2} \simeq 0.40(4)$ $\quad$ `(CKM Fitter ’04)`

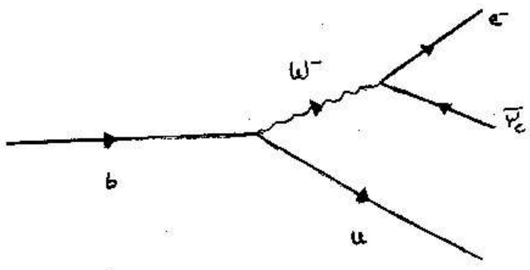
Strategy

- Measure CKM element magnitudes with CP conserving processes
- Measure CKM element phases with CP violating processes
- Impose unitarity conditions and look for inconsistencies

The need for a non-perturbative QCD tool

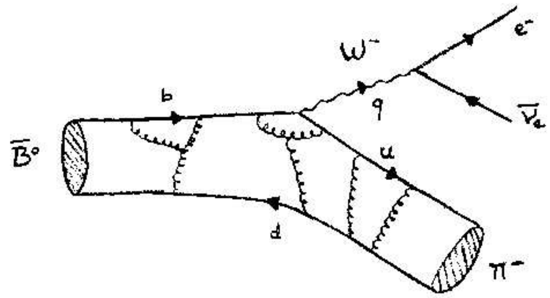
E.g.: exclusive semileptonic b quark decay

At the quark level



$\sim V_{ub} \longrightarrow$

As seen in experiment



$\sim V_{ub}\, \langle \pi^- | \bar{u}\gamma_\mu b | \bar{B}^0 \rangle$

To get model-independent information about $|V_{ub}|$ from experiment:

must evaluate non-perturbative strong interaction corrections in fundamental theory

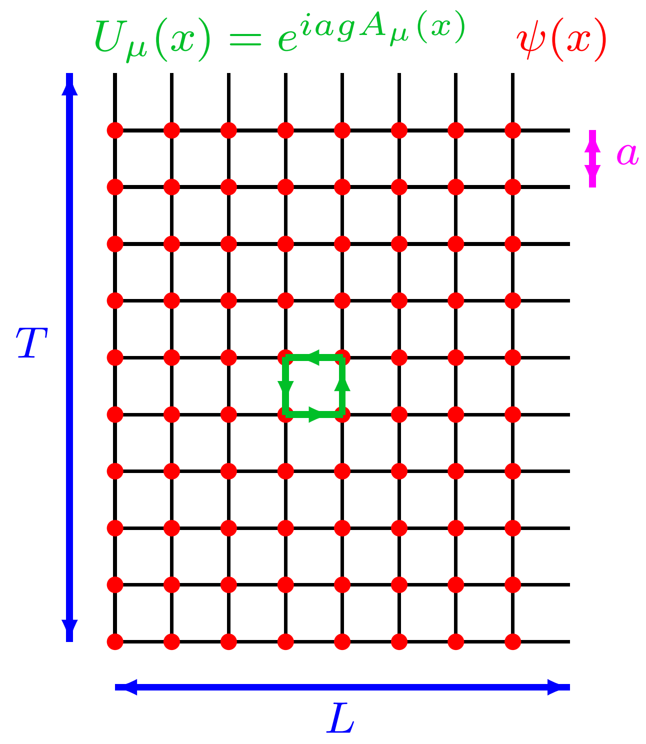
$\Rightarrow$ Lattice QCD

What is lattice QCD (LQCD)?

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G - \int \bar{\psi}D[M]\psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G}\, \det(D[M])\, O[U,\psi,\bar{\psi}]_{\text{Wick}}$$



- $e^{-S_G} \det(D[M]) \geq 0$ and finite # of dof's
$\rightarrow$ evaluate numerically using stochastic methods

NOT A MODEL: LQCD is QCD when $a \to 0$, $V \to \infty$ and stats $\to \infty$

In practice, limitations . . .

Fermions on the lattice

- All (reasonable) discretizations of vector gauge theories have:
  - gauge invariance at finite a
  - Poincaré invariance when $a \to 0$
- Fermion doubling problem makes it very difficult to preserve chiral-flavor symmetries
  $\longrightarrow$ break axial symmetries: Wilson fermions
  $\longrightarrow$ break some of vector and axial symmetries: staggered, twisted-mass fermions
  Symmetries recovered when $a \to 0$ (too much for staggered)
- Full chiral-flavor symmetry at finite a only recently
  $\longrightarrow$ Ginsparg-Wilson fermions (GWF) (domain-wall, Neuberger, overlap . . .) (Kaplan '92, Neuberger '98, Hasenfratz '98, Lüscher '98, ...)
- Faster simulations if we relinquish global symmetries
- GWF numerically more expensive (typically 10-20 times): use when chiral symmetry is crucial

Limitations: finite lattices and quark masses

Limited computer resources $\longrightarrow a$, L and m_q are compromises and statistics finite

Associated errors:

- **Statistical:** $1/\sqrt{N_{conf}}$
- **Discretization:** $a\Lambda_{QCD},\ am_q,\ a|\vec{p}|$, with $a^{-1} \sim 2-4\,\mathrm{GeV}$
  Eliminate with continuum extrapolation $a \longrightarrow 0$

  $1/m_b < a < 1/m_c \Rightarrow b$ quark cannot be simulated directly
  $\longrightarrow$ rely on effective theories (large m_Q expansions of QCD): interpolation between charm and static limit ($m_Q = \infty$) using HQET, FNAL,"NRQCD"
- **Chiral extrapolation:** $m_q \longrightarrow m_u,\ m_d$
  $\longrightarrow$ use χPT to give functional form and chiral logs $\sim M_\pi^2 \ln(M_\pi^2/\Lambda_\chi)$
  Requires $m_q \sim m_s/4 \longrightarrow m_s/8$
- **Finite volume:** for simple quantities, single L large enough that error understood and small
  Eliminate with $L \longrightarrow \infty$ (χPT gives functional form)
- **Renormalization:** LQCD gives bare quantities $\longrightarrow$ must renormalize: can be done in PT, best done non-perturbatively

Limitations: quenching

- Large overhead for computing $\det(D[M])$
  $\longrightarrow$ **quenched approximation** ($N_f = 0$):
  sea quarks are treated as mean field

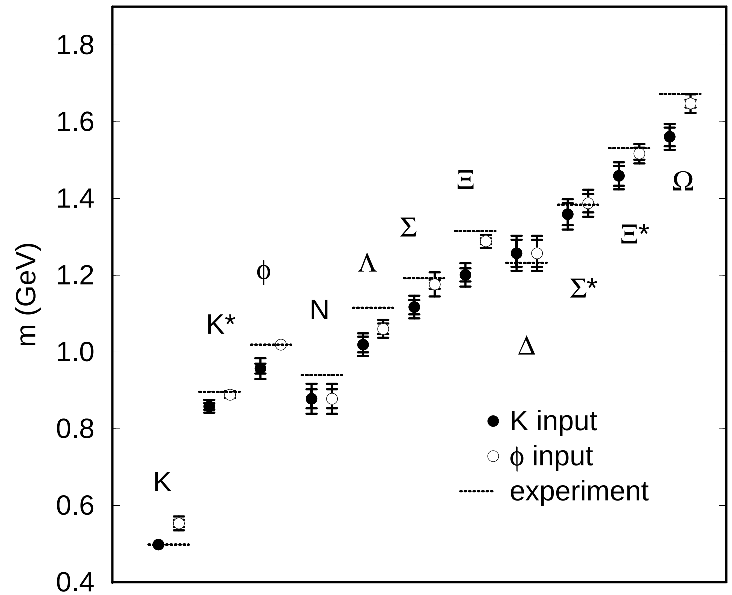
$$\langle O\rangle \approx \int \mathcal{D}U\; e^{-S_G}\, \det(\cancel{D[M]})\; [O]_{\mathrm{Wick}}$$

  $\longrightarrow$ commonly used in past; now used for testing new methods

- Not a systematic appoximation
  Errors $\sim 10-20\%$
  $\longrightarrow$ *Not* QCD but $\sim$ good model

- For many quantities $\longrightarrow$ leading systematic

CP-PACS '02, quenched



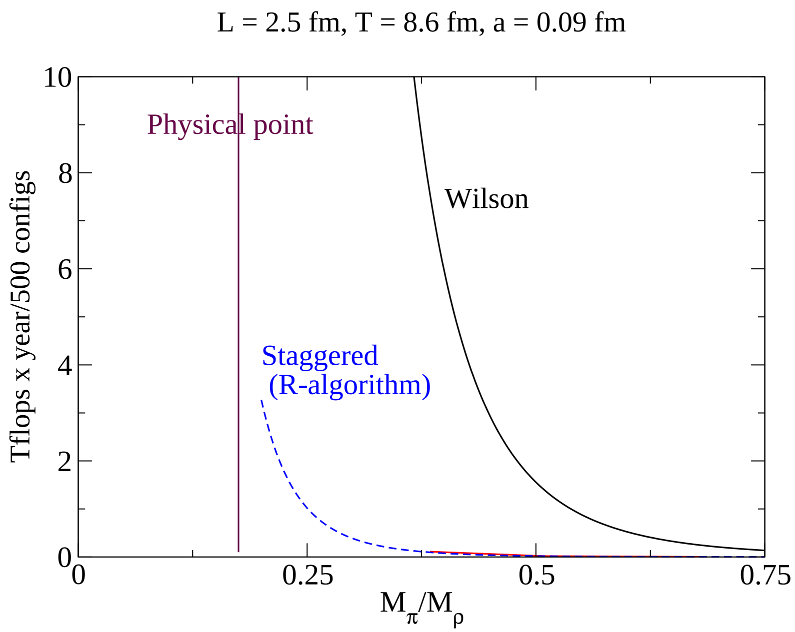
Partial quenching: valence and sea quark masses different

- $N_f = 2$ ($m_s^{\mathrm{sea}} = \infty$): better than quenched

- $N_f = 2+1$: contains QCD ($m^{\mathrm{val.}} = m^{\mathrm{sea}}$); better than real world!

Limitations: the Berlin wall

Unquenched calculations are numerically very demanding: # of d.o.f. $\sim \mathcal{O}(10^9)$



Staggered and Wilson with traditional unquenched algorithms (≤ 2004)

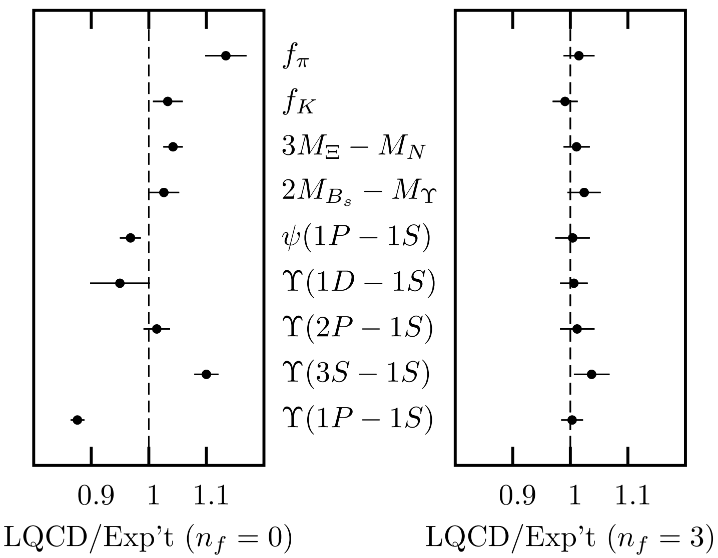
- cost $\sim N_{\rm conf} V^{5/4} m_q^{-2.5\to 3} a^{-7}$ (Gottlieb '02, Ukawa '02)
- Both formulations have a cost wall
- Wall appears for much lighter quarks w/ Staggered, but algorithm not exact

$\longrightarrow$ MILC has gone for the gusto: $N_f = 2+1$ simulations with $m_q \gtrsim 0.1 m_s$!

- Impressive effort: many quantities studied
- Detailed study of chiral extrapolation with staggered χPT (masses small enough)

Are we there?

Certainly looks like it!



(Davies et al '04)

And growing non-perturbative evidence against disaster (Dürr et al '04, ...)

Devil's advocate! $\longrightarrow$ potential problems:

- $\det(D[M])_{N_f=1} \equiv [\det(D[M]_{\text{stagg}})]^{1/4}$ to get rid of spurious "tastes" $\Rightarrow$ no local action known to give that determinant
- algorithm used not exact
- at current a, significant lattice artefacts
- renormalizability of staggered fermions not shown to all orders in PT

$\Rightarrow$ unknown systematic error (is it QCD?)

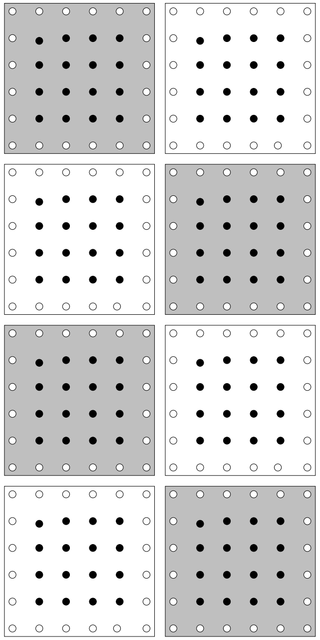
Good testing and learning ground, but *cannot be final word* unless approach is put on firmer ground

The fall of the Berlin wall

Schwarz-preconditioned Hybrid Monte Carlo (SAP) `(Lüscher '03-'04)`

- Applies to Wilson fermions (no $\sqrt[4]{\ }$ trick necessary)
- Decompose lattice into blocks with Dirichlet BCs and
  $p \geq \pi/L \geq 1\,\mathrm{GeV}$
- Asymptotic freedom
  $\longrightarrow$ quark interactions are weak in blocks
  $\longrightarrow$ cheaper simulation
- Block interactions are weak and taken into account exactly
- First principle calculation with

$$\text{cost} \sim N_{\mathrm{conf}} V^{5/4} m_q^{-1} a^{-6}$$



First simulations `(Lüscher '04, Del Debbio et al '05)`: relevant parameter ranges ($m_q \sim m_s/6$, $a^{-1} \sim 3.0\,\mathrm{GeV}$, $= T/2 \sim 2\,\mathrm{fm}$) with ~ 200 Gflops sustained

Limitations: gold-plated quantities

Quantities which the lattice should be able to compute to a few % in the next 5 or so years:

- At most one hadron in initial and final state, e.g.:

$$\sum_{\vec{x}}\langle[\bar{d}\gamma_\mu\gamma_5 u](x)[\bar{u}\gamma_5 d](0)\rangle \stackrel{0\ll t_x\ll T}{\longrightarrow} \langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^+(\vec{0})\rangle\langle\pi^+(\vec{0})|\bar{u}\gamma_5 d|0\rangle\, e^{-M_\pi t_x}$$

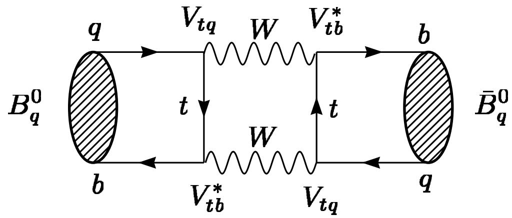
$$\sum_{\vec{x},\vec{y}} e^{i\vec{q}\cdot\vec{y}-i\vec{p}\cdot\vec{x}}\langle[\bar{d}\gamma_5 u](y)[\bar{u}\gamma_\mu b](0)[\bar{b}\gamma_5 d](x)\rangle \stackrel{0\ll t_y\ll T/2\ll t_x\ll T}{\longrightarrow} \langle 0|\bar{d}\gamma_5 u|\pi^+(\vec{q})\rangle\times$$

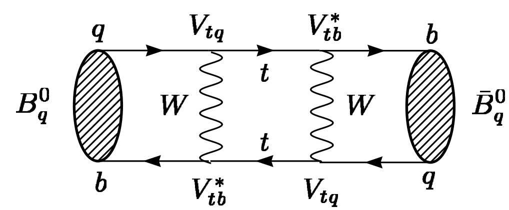
$$\times\langle\pi^+(\vec{q})|\bar{u}\gamma_\mu b|B^0(\vec{p})\rangle\langle B^0(\vec{p})|\bar{b}\gamma_5 u|0\rangle\, e^{-E_B(T-t_x)-E_\pi t_y}$$

- Stable hadrons not near thresholds $\longrightarrow$ ρ, K^*, $\ldots$ difficult
- Disconnected graphs are difficult $\longrightarrow$ no η' (and η due to mixing)
- No high momenta: $a|\vec{p}| \ll 1 \Rightarrow |\vec{p}| \lesssim 1\,\mathrm{GeV}$
- If hadron with momentum requires chiral extrapolation, $|\vec{p}|$ even more limited

$B^0_{(d,s)}-\bar{B}^0_{(d,s)}$ ***mixing***

In Standard Model :





$$\Delta M_q \simeq \frac{G_F^2}{8\pi^2} M_W^2 \, |V_{tq} V_{tb}^*|^2 \, \eta_B S_0(x_t) c_B(\mu) \frac{|\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q \rangle|}{2M_{B_d}}$$

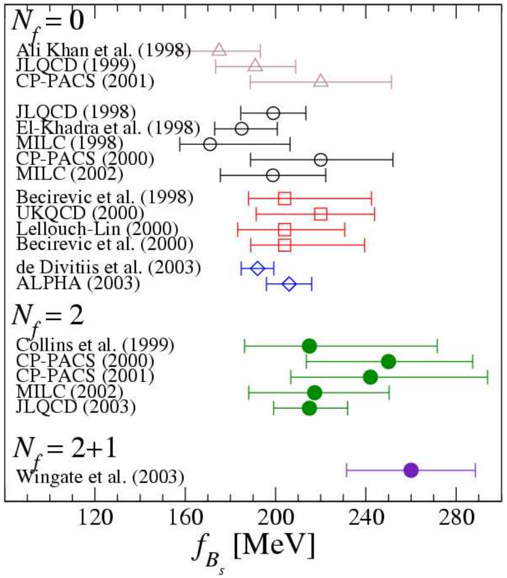
$$= 0.514(5)\,\mathrm{ps}^{-1} \quad [1\%] \qquad \text{for } q = d$$

$$> 14.5\,\mathrm{ps}^{-1} \quad \text{at 95\% CL} \qquad \text{for } q = s$$

$$c_B(\mu) \langle \bar{B}_q^0 | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q^0 \rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 \hat{B}_{B_q}$$

Consider $f_{B_q} = \langle 0 | \bar{b}\gamma_0\gamma_5 q | B_q(\vec{0}) \rangle / M_{B_q}$ and $B_{B_q} = \frac{3\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q \rangle}{8 \langle \bar{B}_q | \bar{b}\gamma_\mu\gamma_5 q | 0 \rangle \langle 0 | \bar{b}\gamma_\mu\gamma_5 q | B_q \rangle}$ separately, because systematics very different

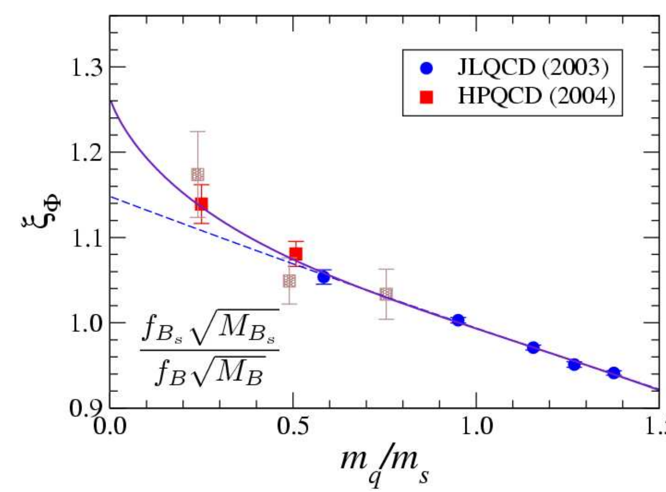
$B^0_{(d,s)} - \bar{B}^0_{(d,s)}$ *mixing: decay constants*



(Hashimoto '04)

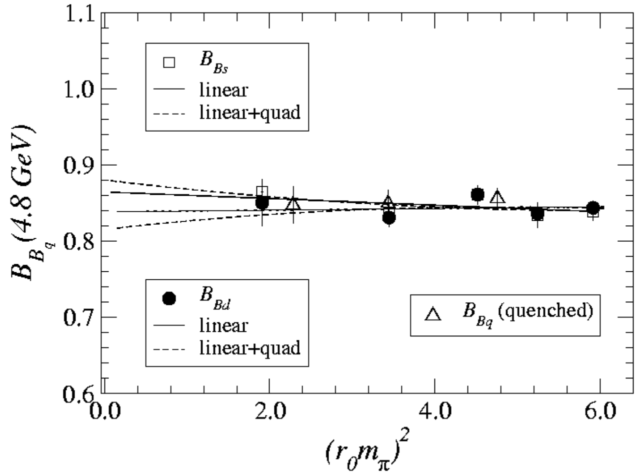
- Dependence of f_{B_s} on m_q only through sea $\to$ weak
- De Divitiis et al '03 and ALPHA '03 perform continuum extrap.
- Different HQ approaches give consistent results in quenched approx.
- Large $N_f = 2+1$ result, but on single rather coarse lattice

- Large chiral log coefficient for f_B (Kronfeld et al '02)
- First hint from preliminary HPQCD '04 results
- Can consider $(f_{B_s}/f_B)/(f_K/f_\pi)$ (Becirevic et al '03) or $(f_{B_s}/f_B)/(f_{D_s}/f_D)$ (Grinstein et al '03) (+ CLEO-c) where logs partially cancel



(Hashimoto '04)

$B^0_{(d,s)}-\bar{B}^0_{(d,s)}$ ***mixing:*** B ***parameters and summary***



$N_f=2$, $m_q \gtrsim m_s/2$ (JLQCD '03)

- Chiral log coefficients small
- Quenching effects appear small, but await $N_f=2+1$ w/ $m_q \lesssim m_s/4$ results
- $\frac{\Delta M_s}{\Delta M_d}=\left|\frac{V_{ts}}{V_{td}}\right|^2\frac{M_{B_s}}{M_B}\xi^2, \quad \xi=\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$
  Many lattice errors cancel, but not chiral log

- Take LL ICHEP '02 and symmetrize chiral log error
- $\delta\xi^2\simeq 12\%$ and $\delta[f_B^2\hat{B}_B]\simeq 30\%$
- Have impact for $\delta[f_B^2\hat{B}_B]\simeq 23\%$ (CKM Fitter '04)
- Non-lattice errors in relation of $\Delta M_s/\Delta M_d$ and ΔM_d to $(\bar\rho,\bar\eta)$ are $\simeq 3\%$ and $\simeq 6\%$, respectively
- Error goals for lattice calculations: $\delta\xi^2\simeq 3\%$ and $\delta[f_B^2\hat{B}_B]\simeq 6\%$

Summary [in MeV]	
f_B	193(27)(10)
f_{B_s}	238(31)
$f_B\hat{B}_B^{1/2}$	223(33)(12)
$f_{B_s}\hat{B}_{B_s}^{1/2}$	276(38)
f_{B_s}/f_B	1.24(4)(6)
ξ	1.24(4)(6)

$|V_{cb}|$ *from* $B \to D^*\ell\nu$

$|V_{cb}|$ plays important rôle in constraining UT $\longrightarrow$ must be determined precisely

- Can extract from differential rate extrapolated to $w = v_B \cdot v_{D^*} = 1$ `(Neubert '91)`

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\nu)\Big|_{w=1} \propto |V_{cb}|^2 |\mathcal{F}_{D^*}(1)|^2$$

- HQET and Luke's theorem give: $\mathcal{F}_{D^*}(1) = 1 + \mathcal{O}(1/m_{c,b}^2)$, but precise measurement of $|V_{cb}|$ requires reliable determination of $\mathcal{F}_{D^*}(1) - 1$

- Double ratios in an $N_f = 0$ calculation (3 a's) `(Kronfeld et al '01, CKM '03)`:

$$\mathcal{F}_{D^*}(1) = 0.913^{+24+17}_{-17-30} \longrightarrow 0.91(4) \qquad [4.4\%]$$

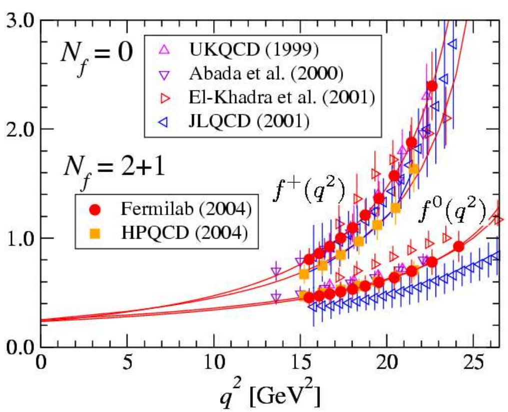
- compare w/ $\delta^{expt.}_{excl.}|V_{cb}| \simeq 2.6\%$, $\delta^{thy}_{incl.}|V_{cb}| \simeq 1.4\%$ and $\delta^{expt.}_{incl.}|V_{cb}| \simeq 1.7\%$

Preliminary $N_f = 2+1$ (MILC) result for $B \to D$ form factor gives $\mathcal{F}_D(1) = 1.075(18)(15)$ `(FNAL '04)`, consistent with $N_f = 0$ result $\mathcal{F}_D(1) = 1.058^{+20}_{-17}$ `(FNAL '99)`

$|V_{ub}|$ **from** $B \to \pi \ell \nu$

In future, best measurement of $|V_{ub}|$ likely to come from exclusive $B \to X_u \ell \nu$

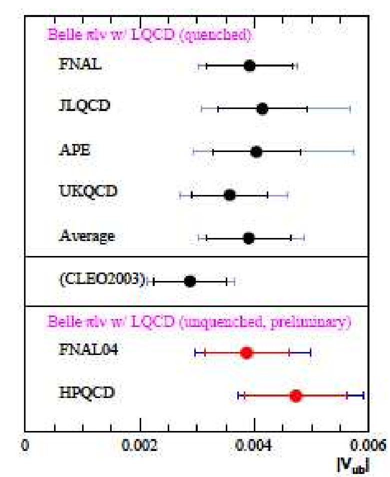
$$\langle \pi^+(\vec{k}) | \bar{u}\gamma_\mu b | \bar{B}^0(\vec{p}) \rangle \longrightarrow f^+(q^2),\ f^0(q^2)$$



(Hashimoto '04)

- Good consistency on $f^+(q^2)$ which determines rate when $m_\ell \to 0$, obtained using different heavy quark approaches
- Errors $\sim O(20\%)$
- $N_f = 2+1$ (MILC) vs $N_f = 0 \Rightarrow$ quenching effects not significant *but* chiral log not yet studied (will limit q^2 more)!

- No model-dependence if directly from differential rate in lattice q^2 range (no HQS normalization here) (Flynn et al '96)
- Belle '04 preliminary ($140\,\mathrm{fb}^{-1}$) from differential rate with quenched LQCD for $q^2 \geq 16\mathrm{GeV}^2$:
$|V_{ub}| = (3.90 \pm 0.71 \pm 0.23^{+0.62}_{-0.48}) \times 10^{-3}$
- $O(10)\times$ statistics expected soon

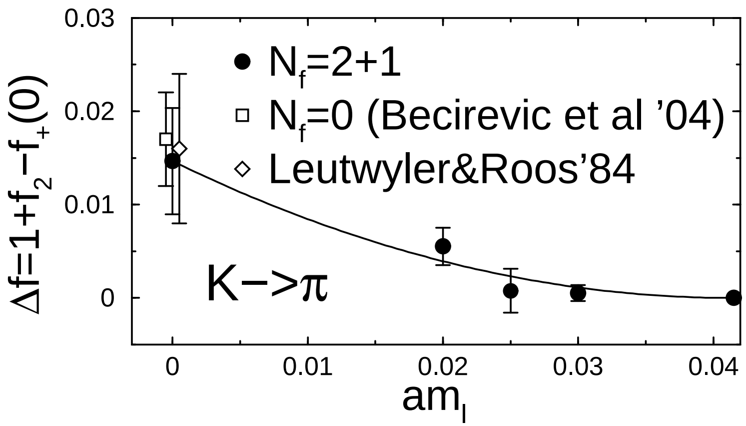


Measurement of $|V_{us}|$ requires theoretical determination of $f^+(0)$ given from:

$$\langle\pi^+(\vec{k})|\bar{u}\gamma_\mu s|\bar{K}^0(\vec{p})\rangle \longrightarrow f^+(q^2),\ f^0(q^2)$$

Becirevic et al '04:

- Calculate $f^0(q^2_{max})$ w/ ratio trick in LQCD
- Use q^2 dependence of $f^0(q^2)$ to obtain $f^+(0) = f^0(0)$ vs $m_l = \bar{m}$
- $\Delta f \equiv 1 + f_2 - f^+(0) \sim (m_s - \bar{m})^2$ by Ademollo-Gatto thm, w/ f_2 the leading logs in χPT
- Preliminary $N_f = 2+1$ (MILC) result from FNAL '04 and $N_f = 2$ result from JLQCD '05

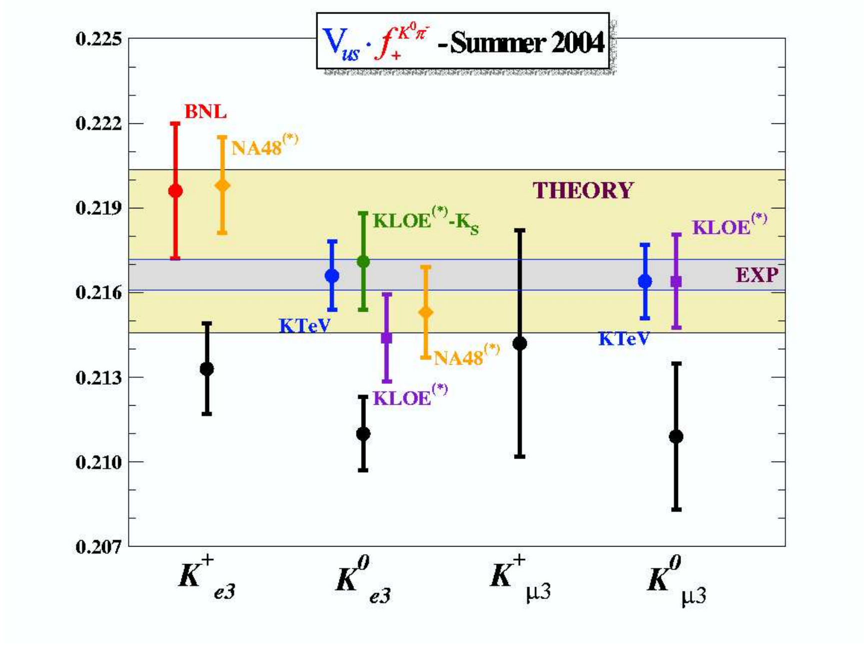


(FNAL '04)

Good agreement with quenched result (Becirevic et al '04), but . . .

. . . MUST verify that $f^+(0)$ from LQCD reproduces NLO and NNLO chiral logs

$|V_{us}|$ ***from*** $K \to \pi\ell\nu$ ***(2)***



- PDG '04: 2σ violation of unitarity relation

  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

- Uses old experimental measurements of $|V_{us}|f^+(0)$ (black points)

- Many new measurements since

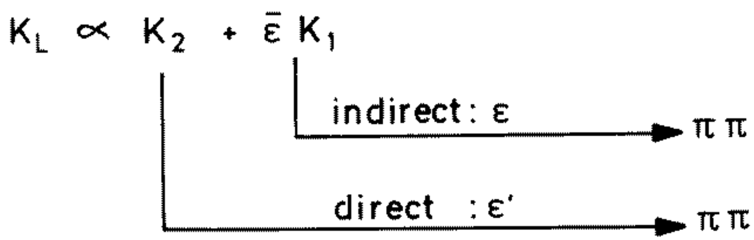
- Good agreement now w/ unitarity relation (theory band, obtained using Leutwyler et al '84)

$K \to \pi\pi$ decays: phenomenology

$$-iT[K^0 \to \pi^+\pi^-] = \sqrt{\tfrac{1}{3}}A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{6}}A_2 e^{i\delta_2} \qquad -iT[K^+ \to \pi^+\pi^0] = \frac{\sqrt{3}}{2}A_2 e^{i\delta_2}$$

$$-iT[K^0 \to \pi^0\pi^0] = -\sqrt{\tfrac{1}{3}}A_0 e^{i\delta_0} + \sqrt{\tfrac{2}{3}}A_2 e^{i\delta_2}$$

CP violation implies $A_I^* \neq A_I$



$$\epsilon \equiv \frac{T[K_L \to (\pi\pi)_{I=0}]}{T[K_S \to (\pi\pi)_{I=0}]} \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\mathrm{Im} M_{12}}{\Delta M_K}$$

$$\epsilon' \simeq \frac{1}{\sqrt{2}} e^{i\pi/4}\, \mathrm{Im}\left(\frac{A_2}{A_0}\right)$$

Experimentally:

$|A_0/A_2| \simeq 22.2$ ($\Delta I = 1/2$ rule)

$|\epsilon| = (2.282 \pm 0.017) \cdot 10^{-3}$ [0.7%]

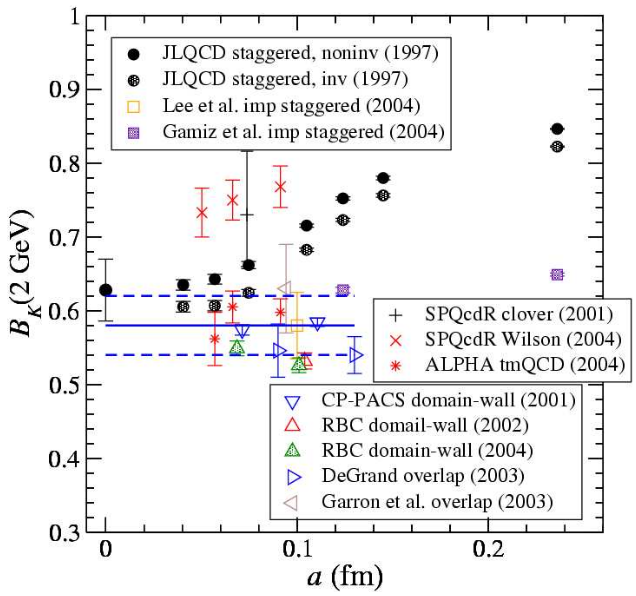
$\mathrm{Re}(\epsilon'/\epsilon) = (16.7 \pm 2.3) \cdot 10^{-4}$ [14%]

Gives constraint on $\mathrm{Im}\left[(V_{ts}^* V_{td})^2\right]$:

$$2M_K M_{12}^* = \langle \bar{K}^0 | H_{\rm eff}^{\Delta S=2} | K^0 \rangle \sim C_K(\mu) \langle \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A}(\mu) | K^0 \rangle$$

$$C_K(\mu) \langle \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A}(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 \hat{B}_K$$

$N_f = 0$



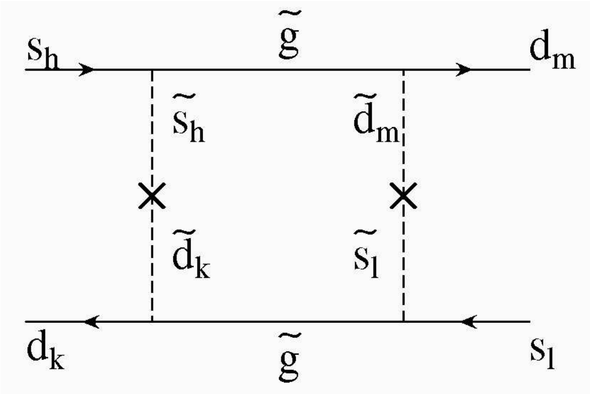
- $N_f = 0$ results consistent in continuum limit $\rightarrow B_K^{NDR}(2\,\mathrm{GeV}) = 0.58(3)$
- Use prelim. DWF, $N_f = 2$, `RBC '05` result with $m_s/2 \lesssim m_q \lesssim m_s$ to estimate quenching (10%) and $m_s \neq m_d$ (5%)

$\rightarrow$ $B_K^{NDR}(2\,\mathrm{GeV}) = 0.58(3)(6)$ or $\hat{B}_K = 0.79(4)(9)$

- $\delta\hat{B}_K \simeq 12\%$ vs non-B_K error in ϵ of 9.5%, mainly from error in $A\lambda^2 = |V_{cb}|$
- Need $N_f = 2+1$ result with light m_q to confirm

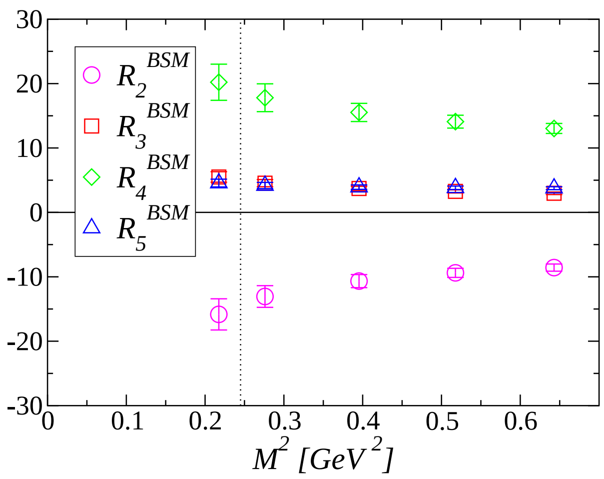
$\Delta S = 2$ processes beyond the SM

e.g. gluino mediated FCNC in mass insertion approximation (Ciuchini et al '99)



$$+\cdots \xrightarrow{OPE} \begin{cases} O_1 = [\bar{s}d]_{V-A}[\bar{s}d]_{V-A} & \\ O_{2,3} = [\bar{s}d]_{S-P}[\bar{s}d]_{S-P} & (\mathrm{unmix}, \mathrm{mix}) \\ O_{4,5} = [\bar{s}d]_{S-P}[\bar{s}d]_{S+P} & (\mathrm{unmix}, \mathrm{mix}) \end{cases}$$

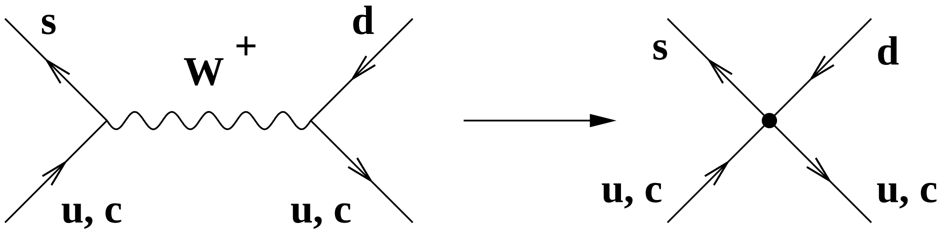
- $N_f = 0$ calculation performed with GWF and NPR (Berruto et al '05)
- $R_i^{\mathrm{BSM}}(M^2) \equiv \left[\frac{F_K^2}{M_K^2}\right]_{expt} \times \left[\frac{M^2}{F^2}\frac{\langle \bar{K}^0|O_i|K^0\rangle}{\langle \bar{K}^0|O_1|K^0\rangle}\right]_{lat}$
- BSM matrix elements are enhanced at M_K by a factor of $\simeq 5 \div 20$
- Enhancement significantly larger than that observed in Wilson fermion calculation of Donini et al '99 $\Rightarrow$ expect improved constraints from $K^0 - \bar{K}^0$ mixing on SM extensions



$\Delta I = 1/2$ rule with an active charm (1)

First step toward a calculation of ϵ' from first principles

OPE (CP-conserving $\Delta S = 1$ transitions)



$$+\text{rad.corr.} \longrightarrow \mathcal{H}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=\pm} C_i(\mu, M_W)\, \tilde{\mathcal{O}}_i(\mu)$$

$$\mathcal{O}_\pm = \Big[(\bar{s}d)_{V-A} (\bar{u}u)_{V-A} \pm (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \Big] - [u \to c]$$

- $\mathcal{O}_-$ is pure $I = 1/2$
- $\mathcal{O}_+$ has both $I = 1/2, 3/2$

Short distance enhancement?

$$|C_-(M_W)/C_+(M_W)| = 1 + \mathcal{O}(\alpha_s(M_W)) \quad \longrightarrow \quad |C_-(2\text{GeV})/C_+(2\text{GeV})| \sim 2$$

(Gaillard et al ’74; Altarelli et al ’74; Shifman et al ’75-’77)

$\Delta I = 1/2$ *rule with an active charm (2)*

Most of enhancement must come from long distance QCD effects in

$$\frac{\langle(\pi\pi)_{I=0}|C_+\mathcal{O}_+ + C_-\mathcal{O}_-|K^0\rangle}{\langle(\pi\pi)_{I=2}|C_+\mathcal{O}_+|K^0\rangle}$$

Some evidence from quenched lattice studies of $K\to\pi$ w/ integrated charm `(CP-PACS '01; RBC '01; Pekurovsky et al '01)`: require subtraction of $1/a^2$ divergences; accurate only to $O(1/m_c^0)$

LQCD w/ χral symmetry $\Rightarrow$ renormalized, continuum operators are given by:

$$\hat{\mathcal{O}}_\pm(\mu) = Z_\pm(\mu a, g_0)\tilde{\mathcal{O}}_\pm(g_0) + \mathcal{O}(a^2)$$

$$\tilde{\mathcal{O}}_\pm = O_\pm + (m_c^2 - m_u^2)C_\pm^m O_m \quad \text{and} \quad \mathcal{O}_m = (m_s+m_d)\bar{s}d - (m_s-m_d)\bar{s}\gamma_5 d$$

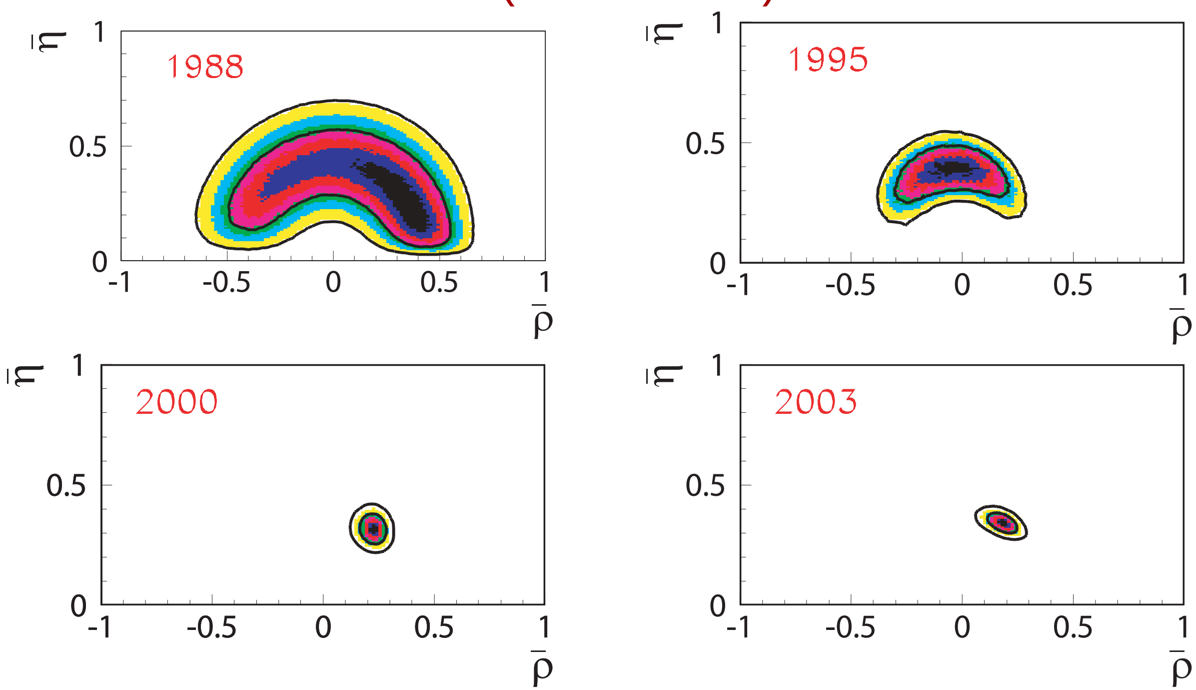
$\Rightarrow$ no power divergent subtractions w/ GW fermions, even when simpler $K\to\pi$ transitions are studied `(e.g. Capitani et al '01)`

$\Rightarrow$ GW fermions provide a unique opportunity to elucidate $\Delta I = 1/2$ puzzle

Conclusion

- Large range of quantities of central importance to particle physics is being computed w/ lattice QCD simulations, many of which could not be presented here
- Experiment and lattice QCD have interacted very positively and will continue to do so in the future

(UTfit '04)



Conclusion (2)

If we are able to simulate full QCD with $O(1-10)$ Tflops with

$$L = 2.0 - 2.5\,\mathrm{fm} \qquad m_q = (0.1 - 0.5)m_s \qquad a = 0.05 - 0.10\,\mathrm{fm}$$

Qty	current err.	goal	projected err.
$f_B^2 \hat{B}_B$	30%	6%	10%
ξ^2	12%	6%	6%
$B \to D^* \ell\nu$	4.4%	1.7%	3%
$B \to \pi\ell\nu$	20%	5%	10%
$f_{K\pi}^+(0)$	1%	0.5%	0.5%
B_K	12%	10%	5%

“Just” need the computer and the man power!

Conclusion (2)

- Dominant systematics are **(partial)-quenching** and **chiral extrapolations** (also discretization errors for HQ) $\rightarrow$ beginning to be addressed head on
- $\Rightarrow$ reliable simulations w/ $N_f = 2+1$ and chiral $m_{u,d}$ will immediately lead to large reduction in systematic error (e.g. $\delta^{quenched}_{syst} f_{\pi,K} \sim 10-15\% \longrightarrow \delta^{N_f=2+1}_{syst} f_{\pi,K} \sim 3\%$! `(MILC '04)`)
- Breakthrough of GW fermions, with their full chiral-flavor symmetry at $a \neq 0 \longrightarrow$ new possibilities for the calculation of weak matrix elements, in particular those associated with the $\Delta I = 1/2$ rule and ϵ'`(cf DWF calculations by CP-PACS and RBC '01; analytical work by Capitani et al '00, '01)`
- Briefly mentioned: reduction in uncertainties obtained by combining ratios of b-quark to equivalent charm quark matrix elements computed on the lattice with charm measurements from e.g. CLEO-c

  $\longrightarrow$ many new $N_f = 2+1$ (MILC) results in the pipeline

Conclusion (3)

Not discussed:

- The finite-volume solution to the longstanding problem of simulating non-leptonic decays such as $K \to \pi\pi$ directly on the lattice (LL et al '01)
- Despite much hard work, difficult to improve on $0.03 < \frac{\Delta\Gamma_s}{\Gamma_s} < 0.15$, at $\sim 3\sigma$, due to large cancellation in leading contribution and large uncertainties in $1/m_b$ correction (e.g. CKM report '03)
- Much work on b-hadron lifetimes (e.g. review by Tarantino, CKM '05)
- GW fermions allow investigation of a numerically unexplored regime of QCD in which the correlation length of pion fields $\gg L$ (ϵ regime of Gasser et al '87)
- In the quenched approximation $\to$ the calculation of low-energy constants (LECs) of the strong chiral lagrangian (Hernández et al '99; DeGrand '01; Hasenfratz '01; Damgaard et al '02; Giusti et al '04)
- It is possible to generalize this approach to extract the LECs of the weak chiral Lagrangian by studying the weak interactions in the ϵ regime (Damgaard et al '02; Giusti et al '02; Hernández et al '02)

Conclusion (4)

- Quenched calculations of the hadronic vacuum polarization relevant for $(g-2)_\mu$, but limited information in dominant kinematical region (Blum ’03, Göckeler et al ’03)
- . . .
- More generally, the range of approaches and the quantities studied on the lattice is constantly expanding $\longrightarrow$ should be many exciting results in years to come