Dynamical twisted mass simulations

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Lattice QCD

Study QCD in a non-perturbative way

- Determine QCD parameters : α_S, $\Lambda_{\rm QCD}$, quark masses
- Determine hadronic properties :
  - Spectrum of mesons and baryons
  - Hadronic structure : form factors, PDF, scattering lengths, ...
- Constrain effective theories :
  - Chiral Perturbation Theory (χPT)
  - Heavy Quark Effective Theory (HQET)
- Constraints on Standard Model parameters : CKM
  - New Physics : precision in the non-perturbative determinations of hadronic matrix elements $\rightsquigarrow$ few percent
  - Control of systematic uncertainties in lattice QCD results

Precision in lattice QCD results

- Control of systematic uncertainties
  - Number of dynamical flavours ($u,d,s,c,\ldots$ quarks) — $N_f = 0;\ 2;\ 2+1;\ 2+1+1$
  - UV cutoff effects: lattice spacing a — $O(a)$ improvement, continuum limit
  - Finite Size (FS) effects: lattice size L — $m_{\rm PS} L \gg 1$
  - Operator renormalisation — non-perturbative
  - Range of quarks masses : simulation/physics — applicability of χPT, HQET
- Statistical errors
  - Improvement in Monte Carlo algorithms — Wilson fermions
  - Supercomputers

Outline

- Simulations with $N_f = 2$ flavours of twisted mass fermions
  - scaling to continuum limit: $\mathcal{O}(a)$ improvement
  - light quark masses: $m_\pi \sim 265$ MeV
  - volumes: $L > 2.1$ fm
- Outline:
  - setup of $N_f = 2$ simulations
  - light-quarks sector
  - strange-quark sector
  - charm-quark sector
  - baryons
  - $N_f = 2+1+1$

twisted mass lattice QCD (tmLQCD): overview

(Frezzotti, Grassi, Sint, Weisz, 1999)

- automatic $\mathcal{O}(a)$ improvement of parity-even correlators guaranteed in maximally twisted lattice QCD (Frezzotti, Rossi, 2003)
  - tuning of only one parameter:
    the bare untwisted quark mass: $m_0 \to M_{\rm cr}$
  - no tuning of operator-specific improvement coefficients
- protection against unphysically small eigenvalues: stability of simulations
- mixing pattern in the renormalization process can be simplified
- low computational cost

But:

- explicit breaking of parity and isospin: the largest cut-off effects are in m_π^0
- however, the breaking is an $\mathcal{O}(a^2)$ effect in physical quantities

tmLQCD: maximal twist

- Twisted basis: $N_f = 2$ flavours

$$S_F^{\mathrm{tmL}} = a^4 \sum_x \bar{\chi}(x) \left[\gamma_\mu \tilde{\nabla}_\mu - r \frac{a}{2} \nabla^*_\mu \nabla_\mu + m_0 + i\gamma_5\tau_3\mu \right] \chi(x)$$

Axial rotation of the quark fields:

$$\psi \to \chi = \exp\left[-i\frac{\omega}{2}\gamma_5\tau_3 \right] \psi \,, \qquad \bar{\psi} \to \bar{\chi}' = \bar{\psi} \exp\left[-i\frac{\omega}{2}\gamma_5\tau_3 \right]$$

$$\text{quark mass} : \quad M_q = \sqrt{\mu^2 + (m_0 - M_{\mathrm{cr}}(r))^2}$$

$$\text{twist angle} : \quad \tan(\omega) = \mu/(m_0 - M_{\mathrm{cr}}(r))$$

- maximal twist: $\omega = \pi/2$
  - untwisted quark mass: $m_q = m_0 - M_{\mathrm{cr}} = 0$
  - twisted mass: $\mu = M_q$

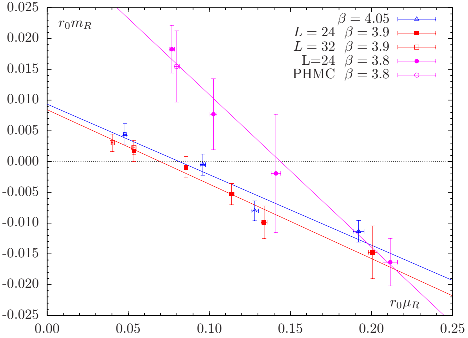
- tuning to maximal twist: several ways to fix $m_0 = M_{\mathrm{cr}}$ $\mathcal{O}(a)$ improvement
  - $m_{\mathrm{PS}} = 0$ at $\mu = 0$
  - $m_{\mathrm{PCAC}} = 0$ at $\mu = 0$
  - $m_{\mathrm{PCAC}} = 0$ at $\mu = \mu_{\mathrm{LOW}}$
  - ...

Tuning to maximal twist

Gauge action: tree level Symanzik

$\beta = 4.05$ $(a \sim 0.066$ fm$)$
$\beta = 3.90$ $(a \sim 0.086$ fm$)$
$\beta = 3.80$ $(a \sim 0.100$ fm$)$

$$m_R \propto m_{\rm PCAC} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(\mathbf{x},t) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(\mathbf{x},t) P^a(0) \rangle} = 0 \qquad \text{at } \mu = \mu_{\rm LOW}$$



$\mu > a^2 \Lambda_{\rm QCD}^3$ $\qquad$ $m_{\rm PCAC} < a\mu\Lambda_{\rm QCD}$

$N_f = 2$

β	target a (fm)	$L^3 \cdot T$	target L (fm)	$a\mu$	N_{traj} $(\tau = 0.5)$	target m_{PS} (MeV)
4.05	$\sim$ 0.066	$32^3 \cdot 64$	2.2	0.0030	5200	$\sim$ 300
				0.0060	5600	$\sim$ 420
				0.0080	5300	$\sim$ 480
				0.0120	5000	$\sim$ 600
		$24^3 \cdot 48$	1.6	0.0060	3000×2	$\sim$ 420
		$20^3 \cdot 48$	1.3	0.0060	5300×2	$\sim$ 420
3.9	$\sim$ 0.086	$24^3 \cdot 48$	2.1	0.0040	10500	$\sim$ 300
				0.0064	5600	$\sim$ 380
				0.0085	5000	$\sim$ 440
				0.0100	5000	$\sim$ 480
				0.0150	5400	$\sim$ 590
		$32^3 \cdot 64$	2.8	0.0030	4500×2	$\sim$ 265
				0.0040	5000	$\sim$ 300
3.8	$\sim$ 0.100	$24^3 \cdot 48$	2.4	0.0060	4700×2	$\sim$ 360
				0.0080	3000×2	$\sim$ 410
				0.0110	2800×2	$\sim$ 480
				0.0165	2600×2	$\sim$ 580
		$20^3 \cdot 48$	2.0	0.0060	4000×2	$\sim$ 360

light-quarks sector

m_π f_π

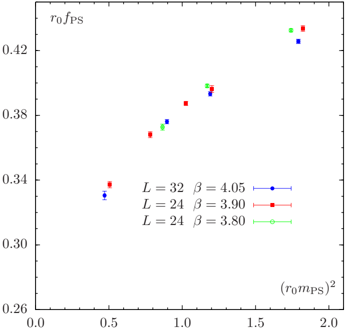
LEC of χPT

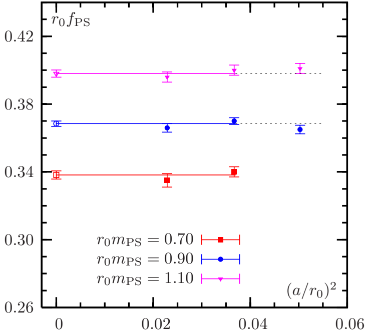
$m_{u,d}$ $\langle \bar{q}q \rangle$

Scaling of f_{PS} at finite volume $\beta = 4.05,\ 3.9,\ 3.8$

$$f_{PS} = \frac{2\mu}{m_{PS}^2} |\langle 0|P^1(0)|\pi^\pm\rangle|$$

$L_{ref} = 2.2$ fm





we use at $\beta = 4.05$: $r_0/a = 6.61(3)$
$\beta = 3.90$: $r_0/a = 5.22(2)$
$\beta = 3.80$: $r_0/a = 4.46(3)$

Continuum estimates and χPT : $f_\pi,\ m_\pi,\ m_N$

Combined fits :

- Combine data at $\beta = 3.9$ and 4.05
- Fit to continuum χPT $+\ O(a^2)$ terms

Continuum extrapolation :

- Bring data to reference volume : $L_{\rm ref} = 2.2$ fm
- Interpolate data points to some reference pion masses :

  $m_{\rm PS} r_0 = (0.7, 0.8, 0.9, 1.0, 1.1, 1.25)$
- Estimate continuum limit by extrapolating at fixed volume :

  Weighted average of data at $\beta = 4.05$ and 3.9

  Use the coarse lattice ($\beta = 3.8$) to estimate systematic error
- Fit to continuum χPT

Chiral perturbation theory : $f_\pi,\ m_\pi,\ m_N$

- Use of χPT to describe the dependence on :
  - the mass μ
  - finite spatial size L

- Simultaneous fit to $N_f = 2$ χPT

$$m^2_{PS}(L) = \chi_\mu \left[1 + \frac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2 \left[1 + \xi \ln(\chi_\mu/\Lambda_3^2) + a^2 D_m\right] ,$$

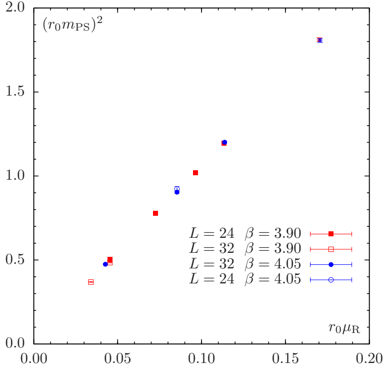
$$f_{PS}(L) = f_0 \ [1 - 2\xi\tilde{g}_1(\lambda)] \left[1 - 2\xi \ln(\chi_\mu/\Lambda_4^2) + a^2 D_f\right] ,$$

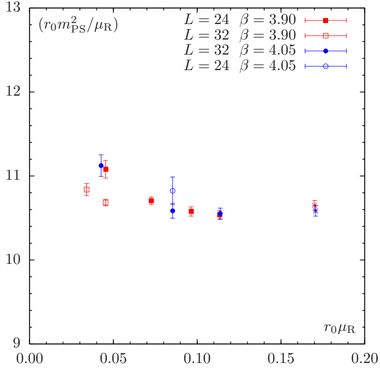
$$m_N(L) = M_N - 4c_1\chi_\mu - \frac{6g_A^2}{32\pi f_0^2}\chi_\mu^{3/2} + a^2 D_N ,$$

where $\chi_\mu = 2B_R\mu_R$, $\xi = \chi_\mu/(4\pi f_0)^2$, $\lambda = \chi_\mu L$, $f_0 = \sqrt{2}F_0$

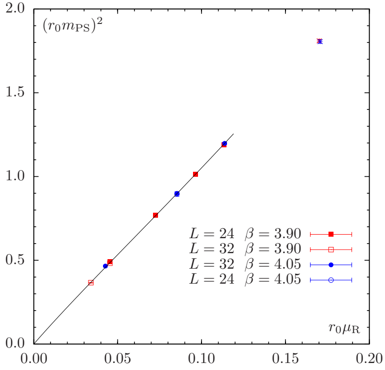
- extract low-energy constants : $\bar{l}_{3,4} \equiv \log(\Lambda^2_{3,4}/m^2_{\pi^\pm})$
- Include $O(a^2)$ terms in the fits
- Finite size corrections : (GL : Gasser, Leutwyler, 1987; CDH : Colangelo *et al.*, 2005)
- Mass dependence : NLO and NNLO (extra parameters : $\Lambda_{1,2}$, k_M, k_F)

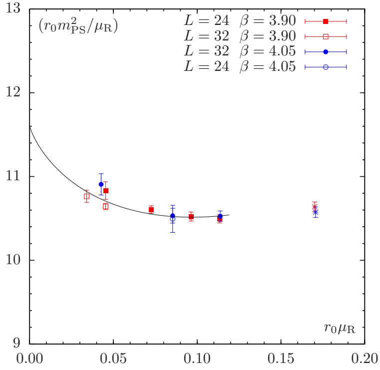
χPT fit : m_{PS}^2 vs. μ_R $\beta = 4.05,\ 3.9$



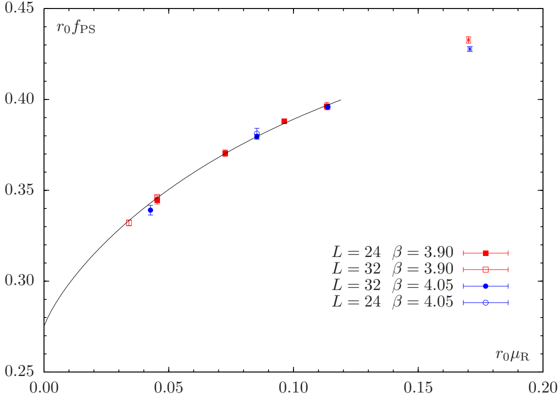


χPT fit : m^2_{PS} vs. μ_R $\beta = 4.05,\ 3.9$





χPT fit : f_{PS} vs. μ_R $\beta = 4.05,\ 3.9$



Combined fit : $O(a^2)$ effects $\beta = 4.05,\ 3.9$

$$r_0 f_{\rm PS} = r_0 f_0 \left[1 - 2\xi \log(\chi_\mu/\Lambda_4^2) + (a/r_0)^2 D_f\right] K_f^{\rm CDH}(L)$$

$$(r_0 m_{\rm PS})^2 = \chi_\mu r_0^2 \left[1 + \xi \log(\chi_\mu/\Lambda_3^2) + (a/r_0)^2 D_m\right] \left(K_m^{\rm CDH}(L)\right)^2$$

$$r_0 m_N = r_0 M_N - 4c_1 \chi_\mu r_0 - \frac{6g_A^2}{32\pi f_0^2 r_0^2}(\chi_\mu r_0^2)^{3/2} + (a/r_0)^2 D_N r_0 M_N$$

- $O(a^2)$ terms : compatible with zero

D_f	D_m	D_N	χ^2/dof
-1.3 ± 1.4	0.5 ± 0.7	0.9 ± 1.1	$16.4/18$

- Other fit parameters $(\Lambda_{3,4}, B_0, \ldots)$ are compatible when including/excluding $O(a^2)$ terms

Combined fit : LEC $\beta = 4.05,\ 3.9$

- fit of f_{PS}, m_{PS} and m_N
- mass dependence : continuum NLO
- volume dependence : CDH

combining $\beta = 4.05$ and 3.9

higher masses ($m_{PS} \sim 600$ MeV) not included

[Colangelo et al., 2005]

- precise results for the χPT low-energy constants : $\bar{l}_{3,4}$

$\bar{l}_3$	3.41(7)
$\bar{l}_4$	4.61(3)
m_N (MeV)	963(47)
$a\|_{\beta=3.9}$ (fm)	0.0855(5)
$a\|_{\beta=4.05}$ (fm)	0.0670(5)
r_0 (fm)	0.445(3)
χ^2/dof	21.4/21

scale :

- fixed using f_π
- reproduces $m_N^{\text{exp.}}$

[Talk by Vincent Drach]

χPT fits : continuum estimates

- fit of f_{PS} and m_{PS}
- mass dependence : continuum NLO
- volume dependence : CDH

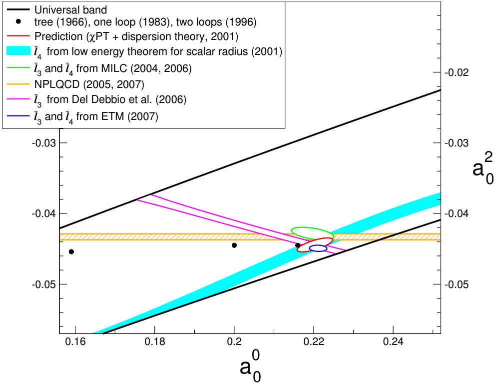
higher masses ($m_{PS} \sim 600$ MeV) not included
[Colangelo et al., 2005]

fit	$\beta = 3.9$	$\beta = 4.05$	combined	continuum extr.
$\bar{l}_3$	3.41(9)	3.66(14)	3.41(7)	3.67(12)(35)
$\bar{l}_4$	4.62(4)	4.66(7)	4.60(3)	4.69(4)(11)
$2r_0\widehat{B}_0$	11.5(3)(4)	12.2(3)(4)	11.6(2)(4)	12.0(3)(6)
$f_0 r_0$	0.275(2)	0.267(5)	0.274(2)	0.266(3)(10)
r_0 [fm]	0.446(4)	0.434(7)	0.445(3)	0.433(5)(16)

Note : "continuum extrap." fit contains fixed volume data ($L \sim 2.2$ fm) whereas "combined fit" includes ensembles from various volumes

$\pi\pi$ scattering

S-wave scattering lengths a_0^0 and a_0^2 [Leutwyler, 2007]



- radius of the scalar pion form factor :
  - This work: $\langle r^2 \rangle_s = 0.64 \pm 0.01 \text{ fm}^2$
  - Colangelo *et. al,* 2001 : $\langle r^2 \rangle_s = 0.61 \pm 0.04 \text{ fm}^2$

Quark masses and chiral condensate

- renormalization constant : non-perturbative Z_P [Talk by Zhaofeng Liu]

- u-d quark mass : combined fit

$$m_{u,d}[\overline{\text{MS}}, 2\ \text{GeV}] = 3.65(6)(23)\ \text{MeV}$$

- chiral condensate :

$$\langle \bar{q}q \rangle = -F_0^2\, \widehat{B}_0$$

combined fit

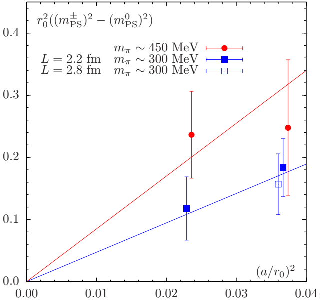
$$(-\langle \bar{q}q \rangle)^{1/3} = 267(1)(7)\ \text{MeV}$$

- comparison: ϵ regime $\beta = 3.9$

$$(-\langle \bar{q}q \rangle)^{1/3} = 264(12)(20)\ \text{MeV}$$

Pion mass splitting

Flavour symmetry is broken at $\mathcal{O}(a^2)$ $\Rightarrow$ $am^0_{PS} \neq am^{\pm}_{PS}$



- difficult measurement : disconnected contributions
- $m^{\pm}_{PS}$, m^0_{PS} mass splitting vanishes like a^2

In the light quark sector, large cut-off effects are only observed in m_{π^0}

- analysis à la Symanzik (Frezzotti, Rossi, 2007)

$$(m^0_{PS})^2 - (m^{\pm}_{PS})^2 = a^2 \zeta^2_\pi + \mathcal{O}(a^2 m^2_\pi, a^4)$$

- $\zeta_\pi = \langle \pi^0 | \mathcal{L}_6 | \pi^0 \rangle |_{\mathrm{cont}}$ has a dynamically large contribution

Isospin splitting

Expect generically large a^2 artifacts all over the place?

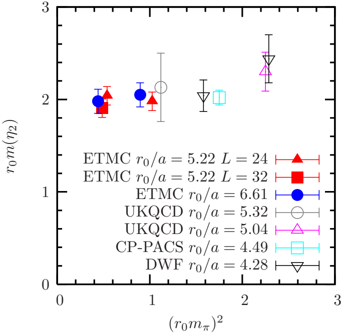
- ζ_π enters only π^0-mass related quantities
- For other observables O :

$$R_O = \frac{O^\pm - O^0}{O^\pm}$$

	β	$a\mu_q$	R_O
af_{PS}	3.90	0.004	0.04(06)
	4.05	0.003	−0.03(06)
am_V	3.90	0.004	0.02(07)
	4.05	0.003	−0.10(11)
af_V	3.90	0.004	−0.07(18)
	4.05	0.003	−0.31(29)
am_Δ	3.90	0.004	0.022(29)
	4.05	0.003	−0.004(45)

- Isospin splittings compatible with zero

Flavour singlet mesons



η_2: flavour singlet pseudoscalar meson in two-flavour QCD

- acquires mass through the $U_A(1)$ anomaly $\rightsquigarrow$ related to $\eta'(958)$
- is expected to have a mass around 800 MeV $\rightsquigarrow$ it is not a Goldstone boson
- chiral extrapolation: consistent with constant behaviour
- $m_{\eta_2} \approx 880$ MeV
- computation requires quark-disconnected diagrams

strange-quark sector

$$m_K \qquad f_K$$

$$m_s$$

$$|V_{us}| \text{ from } K \to \ell\nu$$

strange-quark sector : setup and strategy

- Setup :
  - quark masses (partially quenched)

    $\mu_{\rm sea} = \mu_S$ and $\mu_{\rm val} = \{\mu_1, \mu_2\}$

    - light : μ_S and $\mu_1 \in [1/6\,;\,2/3]\, m_s$
    - strange : $\mu_{1,2} \sim m_s$ (and $\mu_2 \geq \mu_1 = \mu_S$)
  - lattice spacing : $\beta = 3.9$ $a \sim 0.09$ fm
  - volume : $L \sim 2.1$ fm and $m_{\rm PS} L \geq 3.2$
  - statistics : 240 confs for each μ_S
  - stochastic all to all propagators
- Strategy:
  - extrapolation to $m_{u,d}$ and interpolation to m_s
  - experimental inputs:
    - light : $a\mu_{u,d}$ from $(m_\pi/f_\pi)^{\rm exp.}$
    - a from $(f_\pi)^{\rm exp.}$
    - strange : $a\mu_s$ from $(m_K)^{\rm exp.}$

strange-quark sector : mass dependence

chiral perturbation theory (χPT)

- Use of continuum PQχPT to describe the dependence on :
  - the mass μ — NLO (S. Sharpe)
  - finite spatial size L — 1-loop (D. Becirevic and G. Villadoro)
- fit to $N_f = 2$ PQχPT at NLO + "local" NNLO

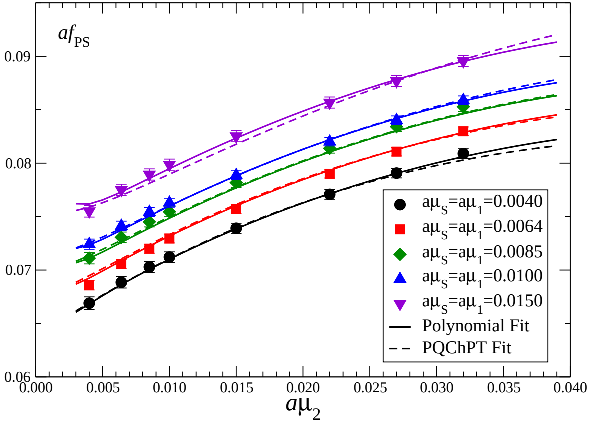
$$m^2_{\rm PS}(\mu_S,\mu_1,\mu_2) = B_0(\mu_1+\mu 2)\left[1+\frac{\xi_1(\xi_S-\xi_1)\ln(2\xi_1)}{\xi_2-\xi_1}-\frac{\xi_2(\xi_S-\xi_2)\ln(2\xi_2)}{\xi_2-\xi_1}+a_V\xi_{12}+a_S\xi_S+ \right.$$
$$\left. +a_{VV}\xi_{12}^2+a_{SS}\xi_S^2+a_{VS}\xi_{12}\xi_S+a_{VD}\xi_{D12}^2\right]$$

$$f_{\rm PS}(\mu_S,\mu_1,\mu_2) = f_0\left[1-\xi_{1S}\ln(2\xi_{1S})-\xi_{2S}\ln(2\xi_{2S})+\frac{\xi_1\xi_2-\xi_S\xi_{12}}{2(\xi_2-\xi_1)}\ln\left(\frac{\xi_1}{\xi 2}\right)+(b_V+1/2)\xi_{12}+\right.$$
$$\left. +(b_S-1/2)\xi_S+b_{VV}\xi_{12}^2+b_{SS}\xi_S^2+b_{VS}\xi_{12}\xi_S+b_{VD}\xi_{D12}^2\right]$$

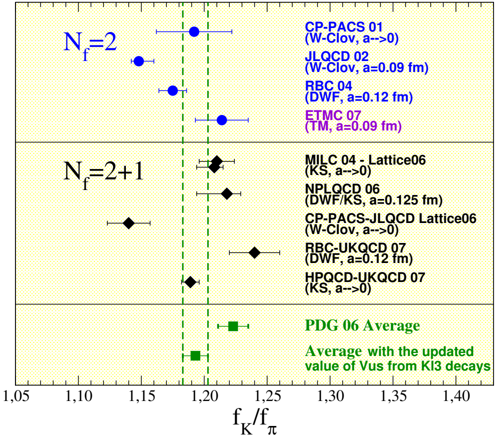
where $\xi_{ij} = B_0(\mu_i+\mu_j)/(4\pi f_0)^2\,,\quad \xi_i=\xi_{ii}\quad \xi_{Dij} = B_0(\mu_i-\mu_j)/(4\pi f_0)^2\,,\quad f_0=\sqrt{2}F_0$

- We also consider polynomial fit functions
- 14 (combined) fit parameters: $B_0, f_0, a_V, a_S, a_{VV}, a_{SS}, a_{VS}, a_{VD}, b_V, \ldots$
- 300 data points: 150 combinations of quark masses (if $\mu_2 \geq \mu_1 = \mu_S$: 30 comb.)

strange-quark sector : $f_{\rm PS}$ vs. μ $\beta = 3.9$



strange-quark sector : f_K/f_π comparison of results

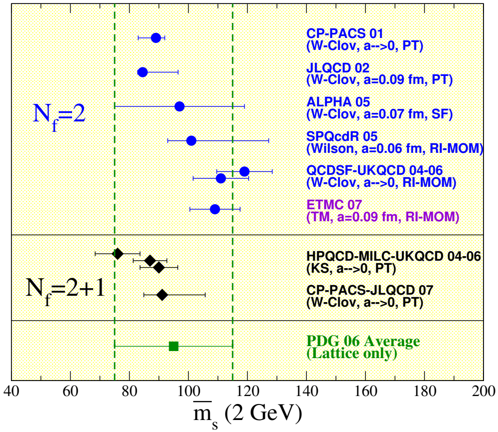


ETMC :

$$f_K = 158.8 \pm 1.3 \pm 2.4 \text{ MeV} \qquad f_K/f_\pi = 1.214(10)(18)$$

$$|V_{us}|/|V_{ud}| = 0.2275(6)(39) \qquad |V_{us}| = 0.2215(5)(38)$$

strange-quark sector : m_s comparison of results



ETMC :

$$m_s[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 109(3)(8)\ \mathrm{MeV} \qquad m_s/m_{u,d} = 27.3(2)(9)$$

charm-quark sector

$$m_D \qquad f_D$$

$$m_{D_s} \qquad f_{D_s}$$

$$m_c$$

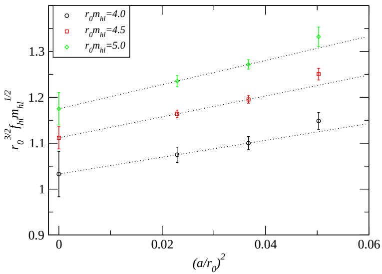
charm sector

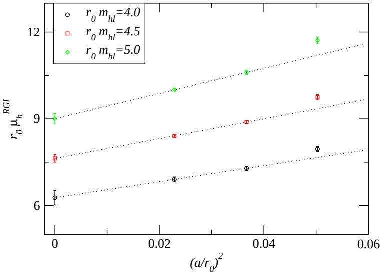
Strategy:

- extrapolation to the continuum limit at fixed physical conditions
- extrapolation to $m_{u,d}$ and interpolations to m_s and m_c
- experimental inputs:
  - light : $r_0\mu_{u,d}$ from $(m_\pi/f_\pi)^{\text{exp.}}$
  - r_0 from $(f_\pi)^{\text{exp.}}$
  - strange : $r_0\mu_s$ from $(m_K)^{\text{exp.}}$
  - charm : $r_0\mu_c$ from $(m_D)^{\text{exp.}}$

charm sector scaling $\beta = 4.05,\ 3.9$ and 3.8

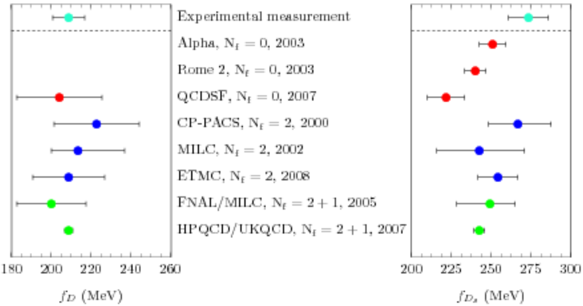
Scaling of $f_D\sqrt{m_D}$ and m_c :





$r_0 m_{\rm PS}(l,l) = 0.9$ $r_0 m_{\rm PS}(l,l) = 0.7$

f_D and f_{D_s}

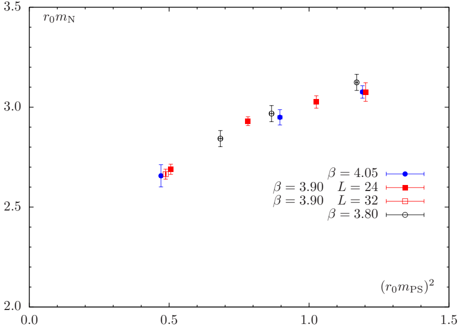


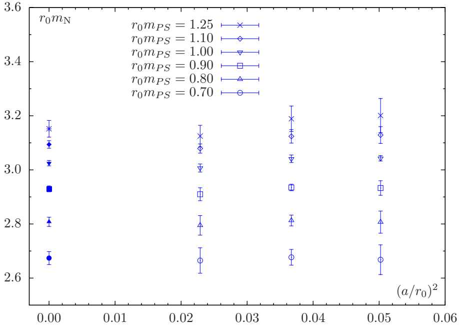
ETMC : continuum estimates

$$f_D = 206(18)(13)\ \mathrm{MeV} \qquad f_{D_s} = 254(8)(10)\ \mathrm{MeV}$$

baryons

nucleon : scaling $\beta = 4.05,\ 3.9$ and 3.8





- cut-off effects appear to be small
- finite volume effects for smallest mass value at $\beta = 3.9$ negligible

baryons : nucleon χPT fits

Chiral extrapolation :

- fitting m_0 and c_1 (continuum HBχPT to 1-loop: $\mathcal{O}(p^3)$)

$$m_\mathrm{N} = m_0 + c_1 m_\mathrm{PS}^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\mathrm{PS}^3$$

- $m_N = 963(12)(8)$ MeV combining $\beta = 3.9$ and 4.05

  $\sigma_N = 66.7 \pm 1.3$ MeV (stat. only)

More results :

- Octet and decuplet spectrum `[talk by Vincent Drach]`
- Nucleon axial coupling g_A `[talk by Rémi Baron]`

$$N_f = 2 + 1 + 1$$

$u, \; d, \; s, \; c$

sea quarks

$N_f = 2+1+1$

- Test QCD in realistic conditions

- Repeat physical conditions of $N_f = 2$ simulations
- Stability of simulations + tuning
- Starting production runs

Action

Combination of :

- Twisted mass lattice action for the light mass degenerate (u-d)-doublet : $N_f = 2$

$$S_{\rm tm} = a^4 \sum_x \{\bar{\chi}_\ell(x) \, [D_W[U] + m_{0\ell} + i\mu_\ell \gamma_5 \tau_3] \, \chi_\ell(x)\} \ ,$$

- Heavy mass non-degenerate (c-s)-doublet term in the action : $N_f = 1+1$

$$\bar{\chi}_h(x) \, [\, i\mu_\sigma \gamma_5 \tau_1 + \mu_\delta \tau_3 \,] \, \chi_h(x)$$

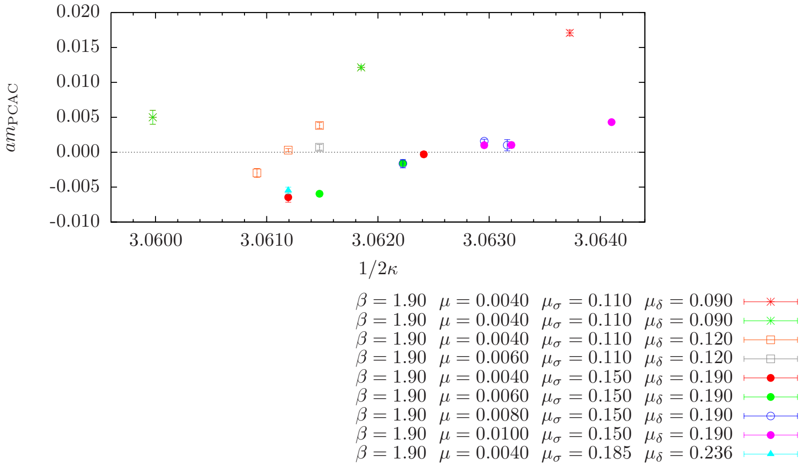
- Automatic $O(a)$ improvement [Frezzotti, Rossi, 2003]

- Non-degenerate quark masses :

$$m_{c,s} = 1/Z_{\rm P} \, \mu_\sigma \pm 1/Z_{\rm s} \, \mu_\delta$$

Tuning to maximal twist

Iwasaki $\beta = 1.9,\ a\mu_l = 0.004,\ L = 24$



$N_f = 2 + 1 + 1$

- Tuning to maximal twist at each μ
- Iwasaki gauge action
- Current runs

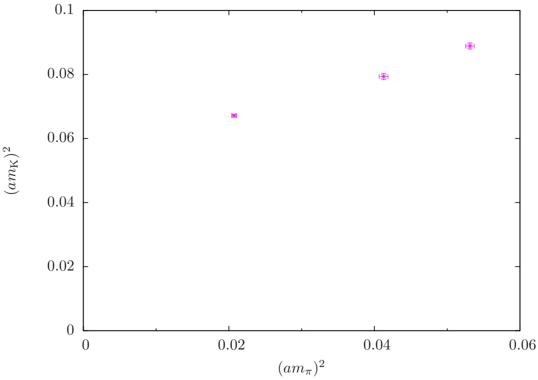
β	$L^3 \cdot T$	$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$
1.90	$24^3 \cdot 48$	0.0040	0.15	0.19
		0.0060		
		0.0080		
		0.0100		
1.95	$32^3 \cdot 64$	0.0035	0.135	0.17

Masses: $m_\pi \in [300; 600]$ Mev $\quad m_K \gtrsim m_K^{\mathrm{exp.}} \quad m_c \gtrsim 10 m_s$

$(am_K)^2$ vs. $(am_\pi)^2$

Iwasaki $\beta = 1.9,\ L = 24,\ a\mu_\sigma = 0.15,\ a\mu_\delta = 0.19$

[PRELIMINARY]

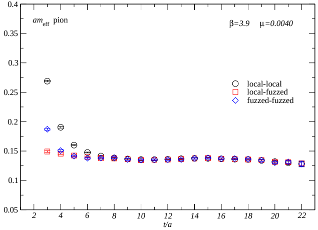


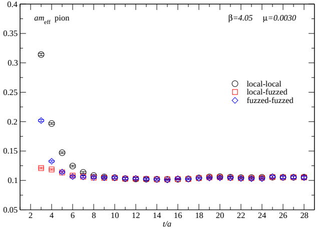
Other ongoing projects ...

- B_K
- meson form factors : pion form factors, semileptonic decays, Isgur-Wise
- static-light mesons
- meson spectrum (ρ, a_0, b_1)
- moments of parton distribution functions
- meson and nucleon scattering phase shifts
- vacuum polarization tensor
- non-perturbative renormalisation
- $N_f = 2+1+1$

Correlators

- Quark propagator: stochastic sources to include all spatial sources
- Change the location of the time-slice source: reduce autocorrelations
- Fuzzing





- effective mass of $\pi^{\pm}$
- isolate ground state : small statistical errors

χPT fits : NLO and NNLO

- check of systematic effects
- fit of $f_{\rm PS}$ and $m_{\rm PS}$
- mass dependence : continuum NLO and NNLO
- volume dependence : CDH

combining $\beta = 4.05$ and 3.9

[Colangelo et al., 2005]

	NLO	NNLO
$\bar{l}_3$	3.41(7)	3.17(18)
$\bar{l}_4$	4.60(3)	4.77(11)
$\widehat{B}_0$ [GeV]	2.57(4)	2.48(5)
f_0 [MeV]	121.6(1)	121.6(2)
r_0 [fm]	0.445(3)	0.448(5)
$\bar{l}_1$	$-0.55(10)$	$-0.63(20)$
$\bar{l}_2$	4.34(1)	4.33(2)
k_M	–	$-0.01(32)$
k_F	–	0.73(33)
χ^2/dof	18.5/16	15.5/16

Input some knowledge on $\bar{l}_{1,2}$, k_M and k_F in the fit:
$\bar{l}_1 = -0.4 \pm 0.6$ $\quad \bar{l}_2 = 4.3 \pm 0.1$ $\quad k_M = k_F = 0 \pm 1$

Finite size effects

Comparison of data at several volumes to :

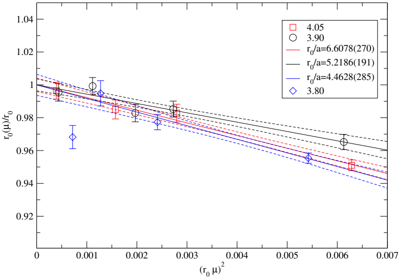
- NLO χPT : GL [Gasser, Leutwyler, 1987, 1988]
- resummed Lüscher formula : CDH [Colangelo, Dürr, Haefeli, 2005]
- relative deviation : $R_O = (O_L - O_\infty)/O_\infty$

obs. O	β	$m_{\rm PS}L$	meas. (%)	GL (%)	CDH (%)
$m_{\rm PS}$	3.90	3.3	+1.8	+0.6	+1.1
$f_{\rm PS}$	3.90	3.3	−2.5	−2.5	−2.4
$m_{\rm PS}$	4.05	3.0	+6.2	+2.2	+6.1
$f_{\rm PS}$	4.05	3.0	−10.7	−8.8	−10.3
$m_{\rm PS}$	4.05	3.5	+1.1	+0.8	+1.5
$f_{\rm PS}$	4.05	3.5	−1.8	−3.4	−2.9

- for $R_{\rm CDH}$: parameters estimates from (CDH, 2005) were used as input
- CDH describes data in general better than GL but needs more parameters

Setting the scale: r_0 vs. μ^2

- Sommer parameter r_0 : static inter-quark force



- HYP-smeared temporal links, APE smeared spatial links, correlator matrix
- statistical accuracy of less than 0.5%,
- compatible with μ^2 dependence

$\Rightarrow$ at $\mu \to 0$: $\beta = 4.05: \ r_0/a = 6.61(3)$ $\beta = 3.9: \ r_0/a = 5.22(2)$
$\beta = 3.8: \ r_0/a = 4.46(3)$

- setting the scale: use several quantities, *e.g.* m_π, f_π, m_K, m_{K*}, f_K, m_N, ...

Pion mass splitting

Expect generically large a^2 artifacts all over the place?

- an analysis a la Symanzik shows that (Frezzotti, Rossi, 2007)

$$(m_{\rm PS}^0)^2 = m_\pi^2 + a^2\zeta_\pi^2 + \mathcal{O}(a^2 m_\pi^2, a^4)\,, \qquad \zeta_\pi \equiv \langle \pi^0|\mathcal{L}_6|\pi^0\rangle|_{\rm cont}$$
$$(m_{\rm PS}^\pm)^2 = m_\pi^2 \qquad\qquad + \mathcal{O}(a^2 m_\pi^2, a^4)$$

- ζ_π has a dynamically large contribution:

$$a^2\zeta_\pi^2 \sim a^2|\hat{G}_\pi|^2\,, \qquad \hat{G}_\pi \equiv \langle 0|\hat{P}^3|\pi^0\rangle = \frac{f_\pi m_\pi^2}{2m_q} \sim (550\,\mathrm{MeV})^2$$

- $|\hat{G}_\pi|^2/\Lambda_{\rm QCD}^4 \sim 25$ $\rightarrow$ potentially large a^2 effects compared to their "natural" size $a^2\Lambda_{\rm QCD}^4$

- ζ_π enters only in quantities related to π^0-mass

$\mathcal{O}(a^2)$ effects in χPT : $f_{\rm PS}$ and $m^2_{\rm PS}$

Power counting for lattice χPT:

$$a \sim \mu \sim m_\pi^2 \sim p^2$$

$\mathcal{O}(a^2)$ effects appear at NNLO only

- $f_\pi|^{\rm Mtm} = f_\pi|^{\rm cont.} \quad + \quad \underset{\text{(NNLO)}}{\mathcal{O}(a^2)} \quad + \quad \underset{\text{(N}^3\text{LO)}}{\mathcal{O}(a^2 m_\pi^2)}$
- $m_\pi^2|^{\rm Mtm} = m_\pi^2|^{\rm cont.} \quad + \quad \underset{\text{(NNLO)}}{\mathcal{O}(a^2 m_\pi^2)} \quad + \quad \underset{\text{(N}^3\text{LO)}}{\mathcal{O}(a^4)}$

Consistent use of continuum χPT at NLO

low-energy constants (LEC)

Accurate determinations of $\bar{l}_{3,4} \equiv \log(\Lambda^2_{3,4}/m^2_{\pi^\pm})$

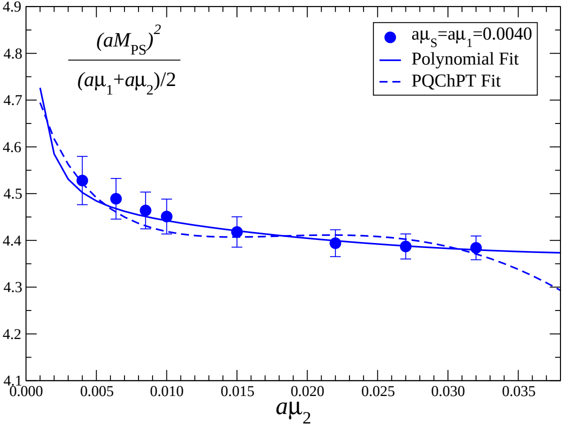
$$\bar{l}_3 = 3.67(12)(35)$$

$$\bar{l}_4 = 4.69(4)(11)$$

Other estimates (Leutwyler, hep-ph/0612112 ; lattice 2007)

- $\bar{l}_3$:
  - $\bar{l}_3 = 2.9 \pm 2.4$ from the mass spectrum of the pseudoscalar octet
  - $\bar{l}_3 = 0.8 \pm 2.3$ from MILC
  - $\bar{l}_3 = 3.0 \pm 0.5$ from CERN
  - $\bar{l}_3 = 3.49 \pm 0.12$ from QCDSF
  - $\bar{l}_3 = 2.9 \pm 0.5$ from JLQCD
  - $\bar{l}_3 = 3.13 \pm 0.33$ from RBC/UKQCD
- $\bar{l}_4$:
  - $\bar{l}_4 = 4.3 \pm 0.9$ from f_K/f_π
  - $\bar{l}_4 = 4.4 \pm 0.2$ from the radius of the scalar pion form factor
  - $\bar{l}_4 = 4.0 \pm 0.6$ from MILC
  - $\bar{l}_4 = 4.69 \pm 0.14$ from QCDSF
  - $\bar{l}_4 = 4.3 \pm 0.6$ from JLQCD
  - $\bar{l}_4 = 4.42 \pm 0.14$ from RBC/UKQCD

strange-quark sector : m^2_{PS}/μ vs. μ $\beta = 3.9$



charm sector : setup and strategy

- Setup :
  - quark masses (partially quenched)

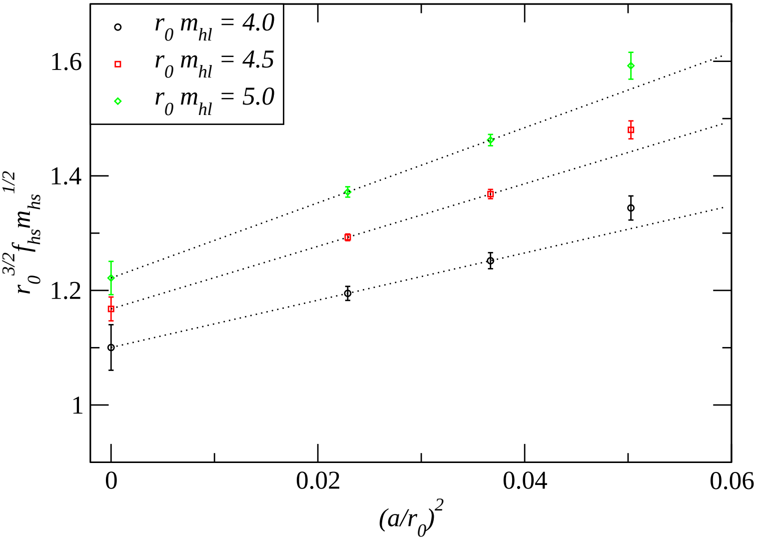
    $\mu_{\rm sea} = \mu_S$ and $\mu_{\rm val} = \{\mu_1, \mu_2\}$

    - light : μ_S and $\mu_1 \in [1/6\,;\,2/3]\, m_s$
    - strange : $\mu_{1,2} \sim m_s$
    - charm : $\mu_2 \sim m_c$
  - lattice spacings : $\beta = 3.8, 3, 9$ and 4.05 $a = (0.07, 0.09, 0.10)$ fm
  - volume : $L \sim 2.1$ fm and $m_{\rm PS} L \geq 3.2$
  - statistics : 230 confs. at $\beta = 3.8$ (each 20 traj. $\tau = 1.0$)

    240 confs. at $\beta = 3.9$ (each 20 traj. $\tau = 0.5$)

    130 confs. at $\beta = 4.05$ (each 40 traj. $\tau = 0.5$)
  - stochastic all to all propagators

Charm sector



Charm sector