Recent results from the BMW collaborabtion

Approaching physical QCD in $2 + 1$ flavor lattice calculations

Budapest-**M**arseille-**W**uppertal Collaboration

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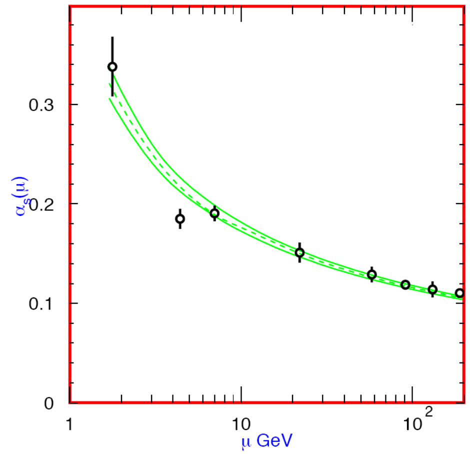
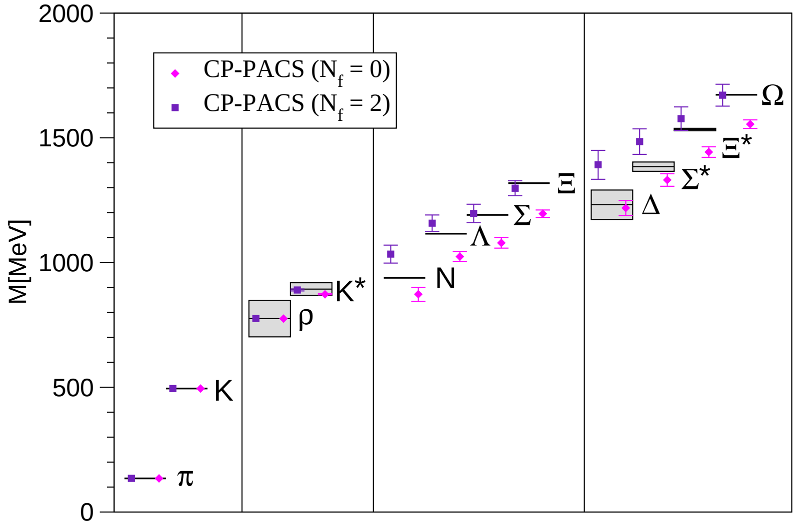
CPT Marseille

QCD: theory of the strong interaction?

QCD well tested at high energies, where it is asymptotically free (PDG '06)



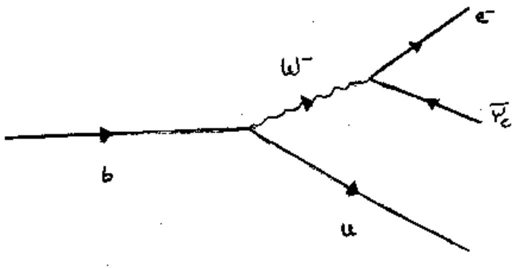


- Good evidence that QCD describes the strong interaction in the non-perturbative domain (e.g. CP-PACS '02 w/ $N_f=2$, $M_\pi \gtrsim 500\,\mathrm{MeV}$, $a \gtrsim 0.11\,\mathrm{fm}$, $L \lesssim 2.6\,\mathrm{fm}$)
- However, systematic errors not under control

Have yet to show agreement (e.g. of hadron *masses* and *widths*) in the physical limit of QCD: $N_f = 2+1$, $M_\pi = 135\,\mathrm{MeV}$, $a \to 0$, $L \to \infty$

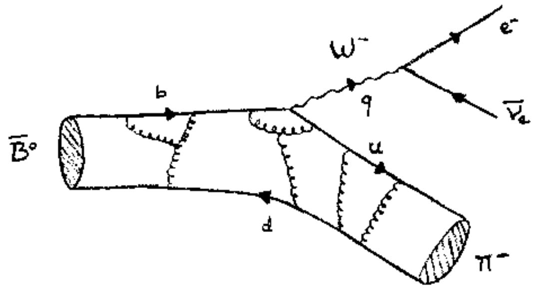
QCD in EW processes

At the quark level



$\sim V_{ub} \longrightarrow$

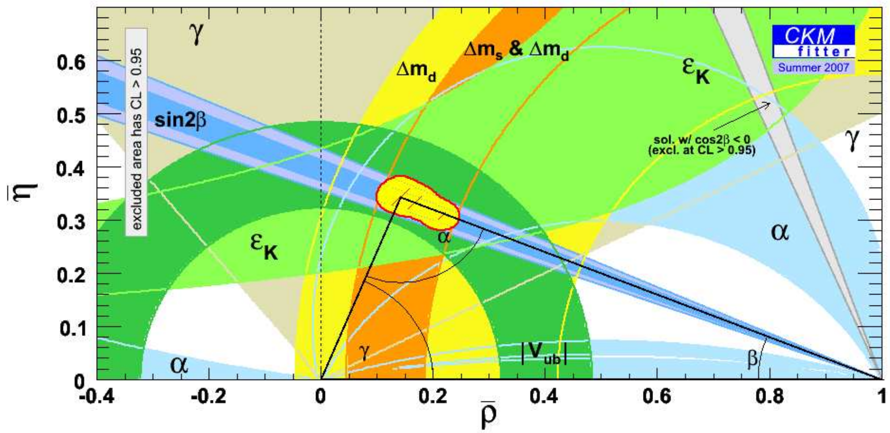
As seen in experiment



$\sim V_{ub}\,\langle \pi^-|\bar{u}\gamma_\mu b|\bar{B}^0\rangle$

$|V_{ub}|$ from experiment $\Rightarrow$ must evaluate non-perturbative strong interaction corrections

- Must be done in QCD to test quark-flavor mixing and CP violation and possibly reveal new physics
- Must match accuracy of BaBar, BELLE, CDF, D0, ALEPH, DELPHI, KLOE, NA48, KTEV, LHC-b, etc.



$\Rightarrow$ High-precision Lattice QCD

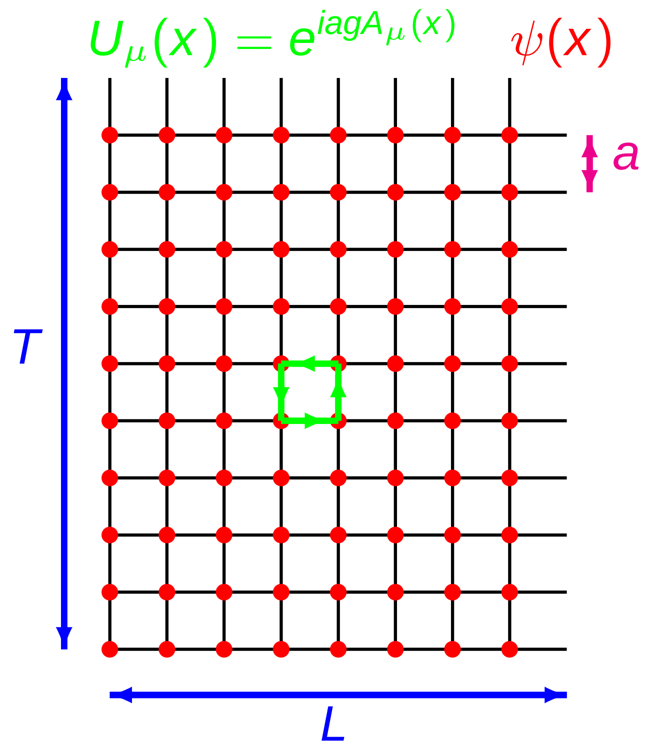
What is Lattice QCD (LQCD)?

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral in Euclidean spacetime:

$$\langle O\rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G-\int\bar{\psi}D[M]\psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G}\, \det(D[M])\, O[U,\psi,\bar{\psi}]_{\rm Wick}$$

- $e^{-S_G}\det(D[M]) \geq 0$ and finite # of dof's $\rightarrow$ evaluate numerically using stochastic methods



NOT A MODEL: LQCD is QCD when $a \rightarrow 0$, $V \rightarrow \infty$ and stats $\rightarrow \infty$

In practice, limitations . . .

Limitations: statistical and systematic errors

Limited computer resources $\rightarrow$ a, L and m_q are compromises and statistics finite

Associated errors:

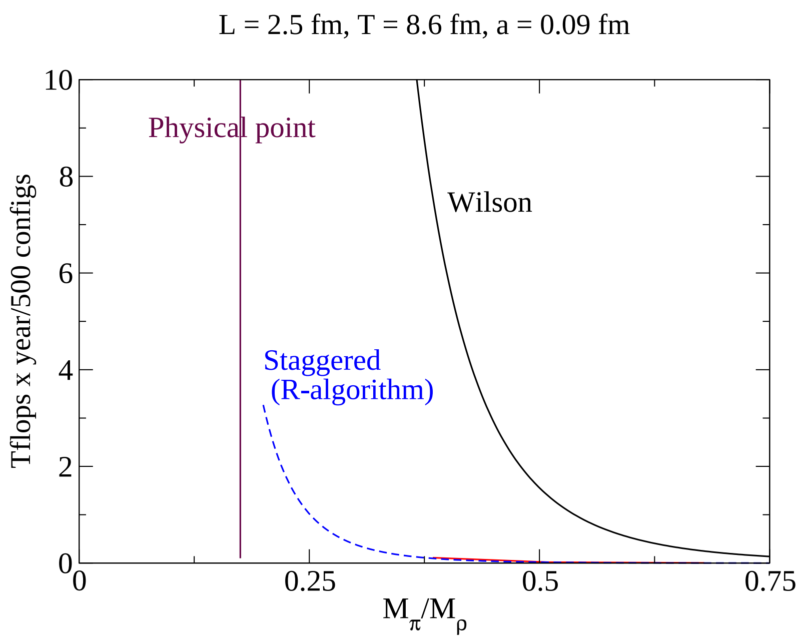
- **Statistical:** $1/\sqrt{N_{conf}}$; eliminate with $N_{conf} \rightarrow \infty$
- **Discretization:** $a\Lambda_{QCD},\ am_q,\ a|\vec{p}|$, with $a^{-1} \sim 2-4\,\mathrm{GeV}$

  $1/m_b < a < 1/m_c \Rightarrow b$ quark cannot be simulated directly
  $\rightarrow$ rely on effective theories (large m_Q expansions of QCD)

  Eliminate with continuum extrapolation $a \rightarrow 0$: need at least three a's
- **Chiral extrapolation:** $m_q \rightarrow m_u,\ m_d$
  Use χPT to give functional form $\rightarrow$ chiral logs $\sim M_\pi^2 \ln(M_\pi^2/\Lambda_\chi)$
  Requires a number of $M_\pi \lesssim 500\,\mathrm{MeV}$
- **Finite volume:** for simple quantities $\sim e^{-M_\pi L}$ and $M_\pi L \gtrsim 4$ usually safe
  Eliminate with $L \rightarrow \infty$ (χPT gives functional form)
- **Renormalization:** LQCD gives bare quantities $\rightarrow$ must renormalize: can be done in PT, best done non-perturbatively

Limitations: the Berlin wall ca. 2001

Unquenched calculations very demanding: # of d.o.f. $\sim \mathcal{O}(10^9)$ and large overhead for computing $\det(D[M])$ ($\sim 10^9 \times 10^9$ matrix) as $m_q \to m_{u,d}$



Staggered and Wilson with traditional unquenched algorithms ($\lesssim$ 2004)

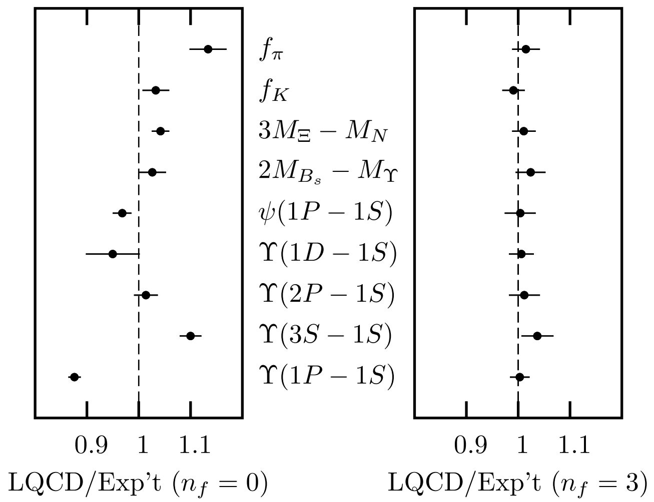
- cost $\sim N_{\text{conf}} V^{5/4} m_q^{-2.5\to 3} a^{-7}$ (Gottlieb '02, Ukawa '02)
- Both formulations have a cost wall
- Wall appears for lighter quarks w/ staggered

$\longrightarrow$ MILC got a head start w/ staggered fermions: $N_f = 2+1$ simulations with $M_\pi \gtrsim 250\,\text{MeV}$

- Impressive effort: many quantities studied
- Detailed study of chiral/continuum extrapolation with staggered χPT

2001 – 2006: staggered dominance and the wall falls

Staggered fermions reign



(Davies et al '04)

Devil's advocate! $\longrightarrow$ potential problems:

- $\det(D[M])_{N_f=1} \equiv \det(D[M]_{\rm stagg})^{1/4}$ to eliminate spurious "tastes"
  $\Rightarrow$ corresponds to non-local theory (Shamir, Bernard, Golterman, Sharpe, 2004-2008)
  $\Rightarrow$ more difficult to argue that $a \to 0$ is QCD
- at current a, significant lattice artefacts
  $\Rightarrow$ complicated chiral extrapolations w/ SχPT
- review of staggered issues in Sharpe '06

$\Rightarrow$ it is important that approaches on firmer theoretical ground also be used

Wilson fermions strike back:

- Schwarz-preconditioned Hybrid Monte Carlo (SAP) (Lüscher '03-'04)
- HMC algorithm with multiple time-scale integration and mass preconditioning (Sexton et al '92, Hasenbusch '01, Urbach et al '06)

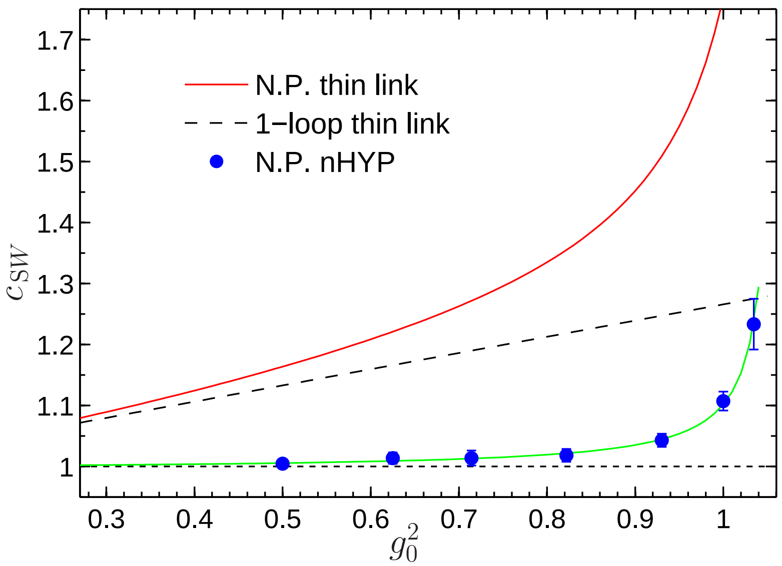
N_f=2+1 Wilson fermions à la BMW

Dürr et al (BMW Coll.) arXiv:0802.2706

- Hasenbusch w/ bells and whistles: RHMC w/ mass preconditioning, multiple time scales, Omelyan integrator and mixed precision techniques
- actions which balance improvements in gauge/fermionic sector and CPU:
  - tree-level $O(a^2)$-improved gauge action (Lüscher et al '85)
  - tree-level $O(a)$-improved Wilson (Sheikholeslami et al '85) with 6-level stout smearing (Morningstar et al '04)

Non-perturbative improvement coefficient c_{SW} close to tree-level value thanks to smearing (Hoffmann et al '07, quenched study w/ nHYP)

$\Rightarrow$ our fermions should be close to being non-perturbatively $O(a)$-improved

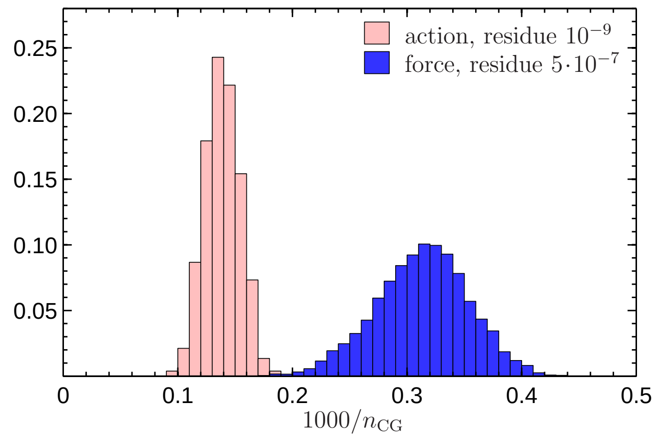


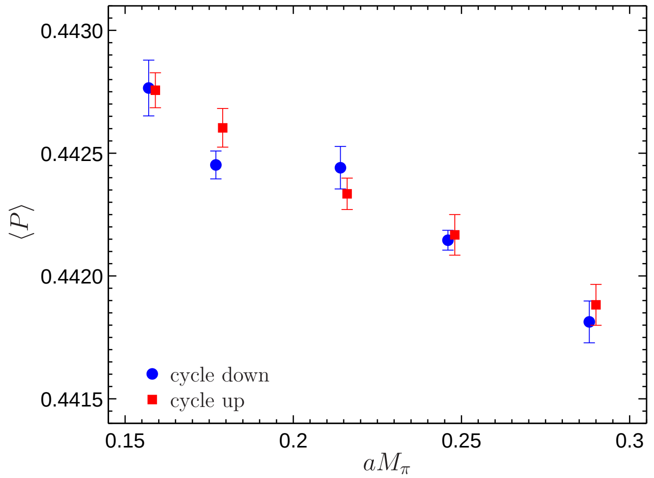
Stability of algorithm

Dürr et al (BMW Coll.) arXiv:0802.2706

Histogram of the inverse iteration number, $1/n_{CG}$, of our linear solver for $N_f = 2+1$, $M_\pi \sim 0.21\,\mathrm{GeV}$ and $L \sim 4\,fm$ (lightest pseudofermion)

Good acceptance





Metastabilities as observed for low M_π and coarse a in Farchioni et al '05?

Plaquette $\langle P \rangle$ cycle in $N_f = 2+1$ simulation w/ $M_\pi \in [0.25, 0.46]\,\mathrm{GeV}$, $a \sim 0.124\,\mathrm{fm}$ and $L \sim 2\,\mathrm{fm}$:

- down from configuration with random links
- up from thermalized config. at $M_\pi \sim 0.25\,\mathrm{GeV}$
- $100 + \sim 300$ trajectories

$\Rightarrow$ no metastabilities observed

$\Rightarrow$ can reach $M_\pi < 200\,\mathrm{MeV}$, $L > 4\,\mathrm{fm}$ and $a < 0.07\,\mathrm{fm}$!

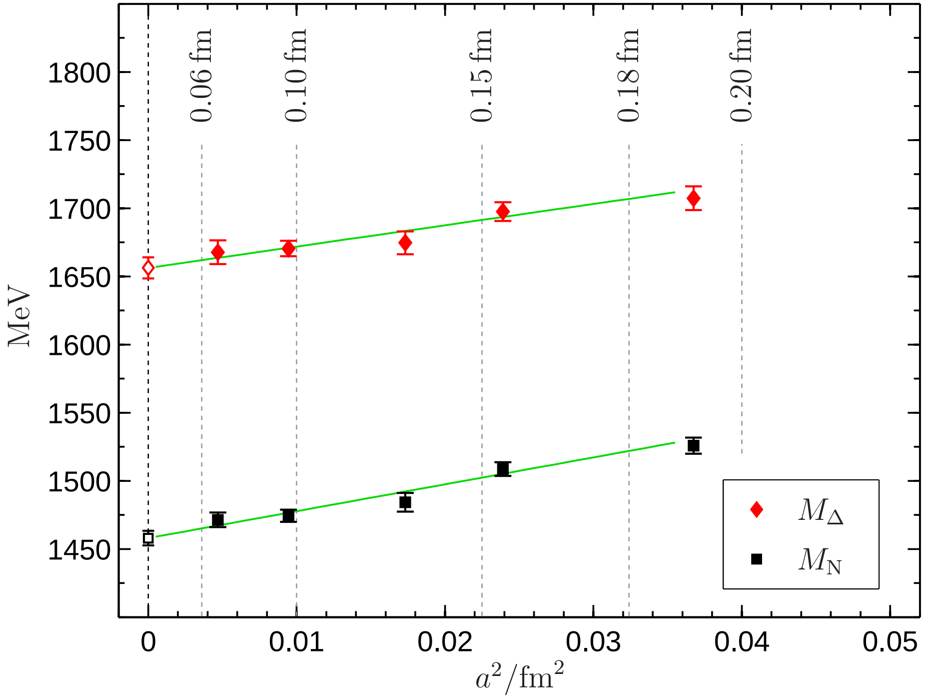
Does our smearing enhance discretization errors?

Dürr et al (BMW Coll.) arXiv:0802.2706

$\Rightarrow$ scaling study: $N_f = 3$ w/ action described above, 5 lattice spacings, $M_\pi L > 4$ fixed and

$$M_\pi/M_\rho = \sqrt{2(M_K^{ph})^2 - (M_\pi^{ph})^2}/M_\phi^{ph} \sim 0.67$$

i.e. $m_q \sim m_s$



M_N and M_Δ are linear in a^2 as a^2 is scaled by a factor 8 up to $a \sim 0.19\,\mathrm{fm}$

$\Rightarrow$ very good scaling

Calculating the light hadron spectrum

Aim: determine the light hadron spectrum in QCD in a calculation in which all systematic errors are controlled

- $\Rightarrow$ **a.** inclusion of sea quark effects w/ an exact $N_f = 2 + 1$ algorithm and w/ an action whose universality class is known to be QCD

  $\longrightarrow$ see above

- $\Rightarrow$ **b.** complete spectrum for the light mesons and octet and decuplet baryons, 3 of which are used to fix m_{ud}, m_s and a
- $\Rightarrow$ **c.** large volumes to guarantee negligible finite-size effects
- $\Rightarrow$ **d.** controlled interpolations to m_s (straightforward) and extrapolations to m_{ud} (difficult)

  Of course, simulating directly around m_{ud} would be better!

- $\Rightarrow$ **e.** controlled extrapolations to the continuum limit: at least 3 a's in the scaling regime

ad b: light hadrons masses and lattice scales

- QCD predicts only ratios of dimensionful qties
  $\Rightarrow$ overall scale can be fixed w/ one mass at the physical point, which should:
  - be calculable precisely
  - have a weak dependence on m_{ud}
  - not decay under the strong interaction

  $\Rightarrow$ 2 good candidates:
  - Ω: largest strange content, but in decuplet
  - Ξ: in octet, but $S{=}-2$

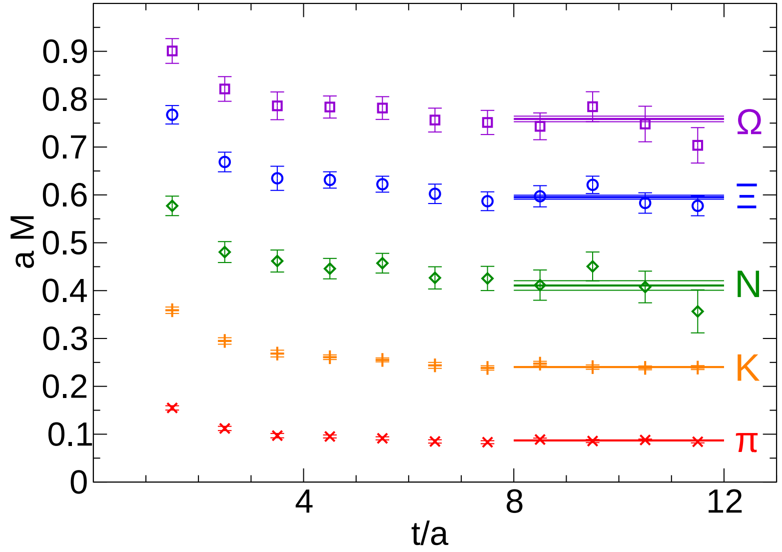
  $\longrightarrow$ 2 separate analyses and compare
- (m_{ud}, m_s) are fixed through: $(M_\pi/M_\Omega, M_K/M_\Omega)$ or $(M_\pi/M_\Xi, M_K/M_\Xi)$
- Determine masses of remaining non-singlet light hadrons:
  - vector meson octet (ρ, K^*)
  - baryon octet $(N, \Lambda, \Sigma, \Xi)$
  - baryon decuplet $(\Delta, \Sigma^*, \Xi^*, \Omega)$

ad b: fits to 2-point functions in different channels

e.g. in pseudoscalar channel, M_π from correlated fit

$$C_{PP}(t) \equiv \frac{1}{(L/a)^3} \sum_{\vec{x}} \langle [\bar{d}\gamma_5 u](x) [\bar{u}\gamma_5 d](0)\rangle \xrightarrow{0 \ll t \ll T} \frac{\langle 0|\bar{d}\gamma_5 u|\pi^+(\vec{0})\rangle\langle\pi^+(\vec{0})|\bar{u}\gamma_5 d|0\rangle}{2M_\pi} e^{-M_\pi t}$$

- Effective masses $aM(t+a/2) = \log[C(t)/C(t+a)]$ for our simulation at $a \approx 0.082\,\mathrm{fm}$ and $M_\pi \approx 0.21\,\mathrm{GeV}$
- Gaussian sources and sinks with $r \sim 0.32\,\mathrm{fm}$



Resonances are treated as stable particles for now:

- all are ground state, except the ρ's for $a \approx 0.082\,\mathrm{fm}$ and $M_\pi \approx 0.19,\ 0.21\,\mathrm{GeV}$ (ρ unstable in $L \to \infty$ for $M_\pi \lesssim 0.44\,\mathrm{GeV}$)
- ρ source/sink are $\bar{u}\gamma_i d$ operators which have $\sim 1/(N_c L^3)$ suppressed overlap with scattering $\pi\pi$ states
  $\Rightarrow$ only ρ contributes for fitted times

ad c: infinite-volume limit

- For stable particles in large volumes $\mathrm{FVE} \sim e^{-M_\pi L}$
- $M_\pi L \gtrsim 4$ expected to give $L \to \infty$ masses within our statistical errors
- For $a \approx 0.124\,\mathrm{fm}$ and $M_\pi \approx 0.39\,\mathrm{GeV}$, found $M_\pi L \sim 8$ results compatible w/ $M_\pi L \sim 4$ results

Simulation parameters

β, a [fm]	am_{ud}	M_π [GeV]	am_s	$L^3 \times T$	# traj.
3.3	-0.0960	0.66	-0.057	$16^3 \times 32$	10000
	-0.1100	0.52	-0.057	$16^3 \times 32$	1450
$\approx$ 0.124	-0.1200	0.39	-0.057	$16^3 \times 64$	4500
	-0.1233	0.34	-0.057	$24^3 \times 64$	600
	-0.1265	0.28	-0.057	$24^3 \times 64$	700
3.57	-0.03175	0.53	0.0	$24^3 \times 64$	1250
	-0.03175	0.52	-0.01	$24^3 \times 64$	1650
$\approx$ 0.082	-0.03803	0.44	0.0	$24^3 \times 64$	1300
	-0.03803	0.43	-0.01	$24^3 \times 64$	1300
	-0.044	0.32	0.0	$32^3 \times 64$	1000
	-0.044	0.32	-0.07	$32^3 \times 64$	1000
	-0.0483	0.21	0.0	$48^3 \times 64$	500
	-0.0483	0.19	-0.07	$48^3 \times 64$	1000
3.7	-0.007	0.64	0.0	$32^3 \times 96$	600
	-0.013	0.55	0.0	$32^3 \times 96$	500
$\approx$ 0.065	-0.02	0.43	0.0	$32^3 \times 96$	700
	-0.022	0.39	0.0	$32^3 \times 96$	600
	-0.025	0.31	0.0	$40^3 \times 96$	500

- # of trajectories given is after thermalization
- autocorrelation times less than $\approx$ 10 trajectories
- 2 runs with 10000 and 4500 trajectories
  $\longrightarrow$ no long-range correlations found

ad d: extrapolation to m_{ud} and interpolation to m_s

Assume here that scale is set by M_Ξ; analogous expressions hold when scale is set by M_Ω:

- $R_X \equiv M_X/M_\Xi$ can be viewed as fns of aM_Ξ, R_π and R_K
- physical QCD point reached for $R_\pi \to R_\pi^{ph}$, $R_K \to R_K^{ph}$ and $aM_\Xi(R_\pi^{ph}, R_K^{ph}) \to 0$
- linear term in R_K^2 is sufficient for interpolation to m_s
- curvature in R_π^2 is visible in extrapolation to m_{ud} in some channels
- write R_X as Taylor expansion in M_π and M_K around physical point:

$$R_X = R_X^{ph} + \alpha_X[R_\pi^2 - R_\pi^{ph,2}] + \beta_X[R_K^2 - R_K^{ph,2}] + \mathrm{hot}$$

$\longrightarrow$ χPT suggests M_π^3 NLO behavior (Langacker et al ’74)
$\longrightarrow$ in Taylor expansion $\mathrm{hot} \sim M_\pi^4$

$\Rightarrow$ try each or none and use differences as systematic error

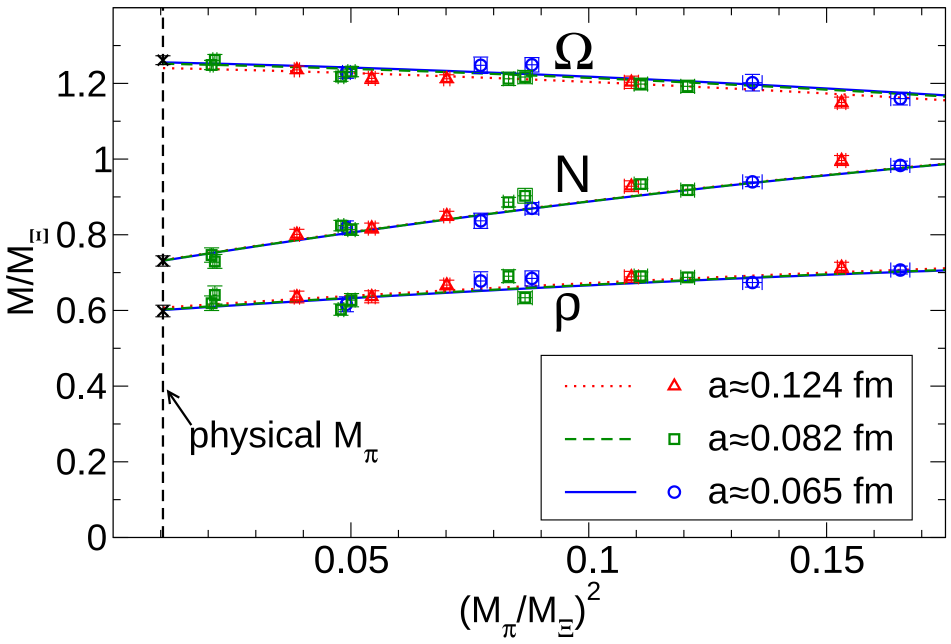
- Input: R_X, R_π and R_K $\longrightarrow$ primary output R_X^{ph}

ad e: including continuum extrapolation

Cutoff effects:

$$R_X^{ph} \longrightarrow R_X^{ph}[1+\gamma_X\, a] \quad \text{or} \quad R_X^{ph}[1+\gamma_X\, a^2]$$

$\Rightarrow$ not sensitive to am_s or am_{ud}



Systematic errors:

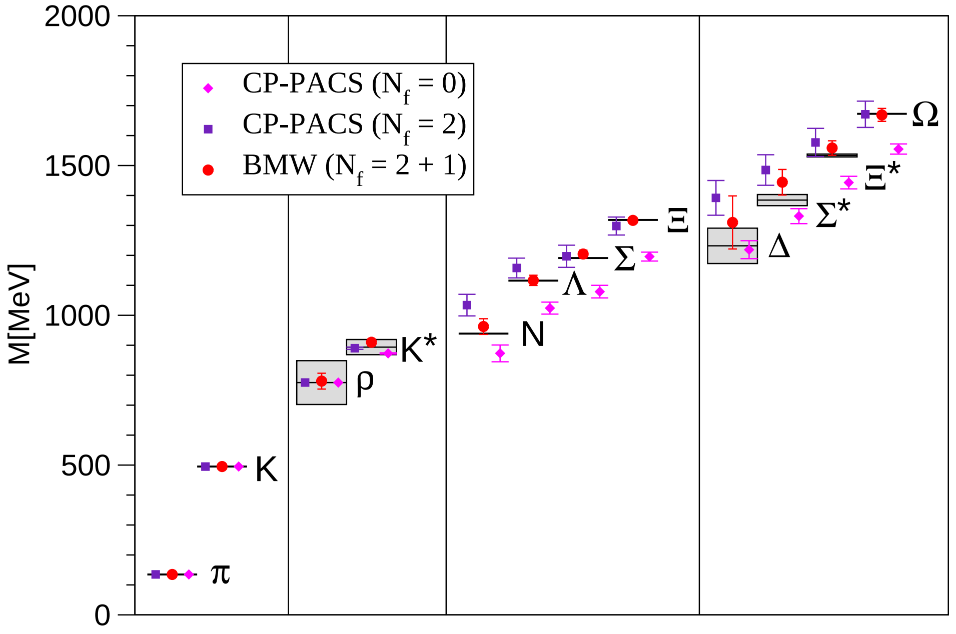
- correlator fits: 30 different combinations of time intervals
- mass-dependence fits: purely quadratic or an additional M_π^3 (χPT) or M_π^4 (Taylor) dependence
- continuum extrapolation: a^0, a or a^2 terms in R_X^{ph}

$\Rightarrow 30 \times 3 \times 3 = 270$ different results for M_X^{ph}

Statistical errors $\longrightarrow$ bootstrap (2000 samples)

For both, take central 68%

Post-dictions for the light hadron spectrum



Results in GeV with statistical/systematic errors

X	M_X^{expt}	M_X (Ω set)	M_X (Ξ set)
ρ	0.775	0.773(27)(28)	0.780(20)(15)
K^*	0.894	0.909(14)(8)	0.910(9)(5)
N	0.939	0.958(16)(28)	0.963(14)(21)
Λ	1.116	1.112(14)(23)	1.117(10)(13)
Σ	1.191	1.202(17)(12)	1.205(12)(2)
Ξ	1.318	1.318(14)(11)	1.318
Δ	1.232	1.318(57)(80)	1.310(51)(72)
Σ^*	1.385	1.452(34)(29)	1.444(29)(30)
Ξ^*	1.533	1.565(25)(10)	1.558(17)(16)
Ω	1.672	1.672	1.669(15)(14)

- results from Ξ and Ω sets perfectly consistent
- errors smaller in Ξ set
- agreement with experiment is excellent

$|V_{us}|$ from $K \to \mu\bar{\nu}$

Precision tests of SM and constraints on new physics (NP) from

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$$

Currently

- $|V_{ud}| = 0.97418(26)$ [0.03%] from nuclear β decays (Hardy & Towner '07)
  $\Rightarrow \delta|V_{ud}|^2 = 5.2 \cdot 10^{-4}$
- $|V_{us}| = 0.2246(12)$ [0.5%] from K_{l3} (Flavianet '07)
  $\Rightarrow \delta|V_{us}|^2 = 2.4 \cdot 10^{-3}$
- $|V_{ub}| = 3.86(9)(47) \cdot 10^{-3}$ (CKMfitter '07)
  $\Rightarrow |V_{ub}|^2 \simeq 10^{-5}$

$\Rightarrow$ dominant uncertainty from $|V_{us}|$

$\Rightarrow \Lambda_{NP} \gtrsim 2\,\mathrm{TeV}$

$|V_{us}|$ from $K \longrightarrow \mu\bar{\nu}$

Marciano '04: window of opportunity

$$\frac{\Gamma(K \to \mu\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))} \longrightarrow \frac{|V_{us}|}{|V_{ud}|}\frac{F_K}{F_\pi} = 0.2760(6)\ [0.22\%]$$

Error is smaller than current error on $|V_{us}|$ from $K \to \pi\ell\nu$!

Need:

- F_K/F_π to 0.5% to match $K \to \pi\ell\nu$ determination (assuming that systematics in that determination are controlled to that level)
- F_K/F_π to 0.22% to match experimental error in $K \to \mu\bar{\nu}(\gamma)/\pi \to \mu\bar{\nu}(\gamma)$

On lattice, get e.g. F_K from

$$C_{A_0P}(t) \equiv \frac{1}{(L/a)^3}\sum_{\vec{x}}\langle[\bar{s}\gamma_5\gamma_0 u](x)[\bar{u}\gamma_5 s](0)\rangle \overset{0\ll t\ll T}{\longrightarrow} \frac{\langle 0|\bar{s}\gamma_5\gamma_0 u|K^+(\vec{0})\rangle\langle K^+(\vec{0})|\bar{u}\gamma_5 d|0\rangle}{2M_K}e^{-M_K t}$$

and

$$\langle 0|\bar{s}\gamma_5\gamma_0 u|K^+(\vec{0})\rangle = \sqrt{2}M_K F_K$$

F_K/F_π from the lattice: preliminary results

Results for F_K/F_π are corrected for small FV effects using 2-loop χPT (Colangelo et al '05)

Mass dependence of F_K/F_π studied w/ NLO $SU(3)$ χPT, allowing for $O(p^4)$ analytic terms

$$\frac{F_K}{F_\pi} = 1 + \frac{1}{32\pi^2 F_0^2}\left\{\frac{5}{4}M_\pi^2 \log(\frac{M_\pi^2}{\mu^2}) - \frac{1}{2}M_K^2 \log(\frac{M_K^2}{\mu^2}) - [M_K^2 - \frac{1}{4}M_\pi^2]\log(\frac{4M_K^2 - M_\pi^2}{3\mu^2})\right\}$$

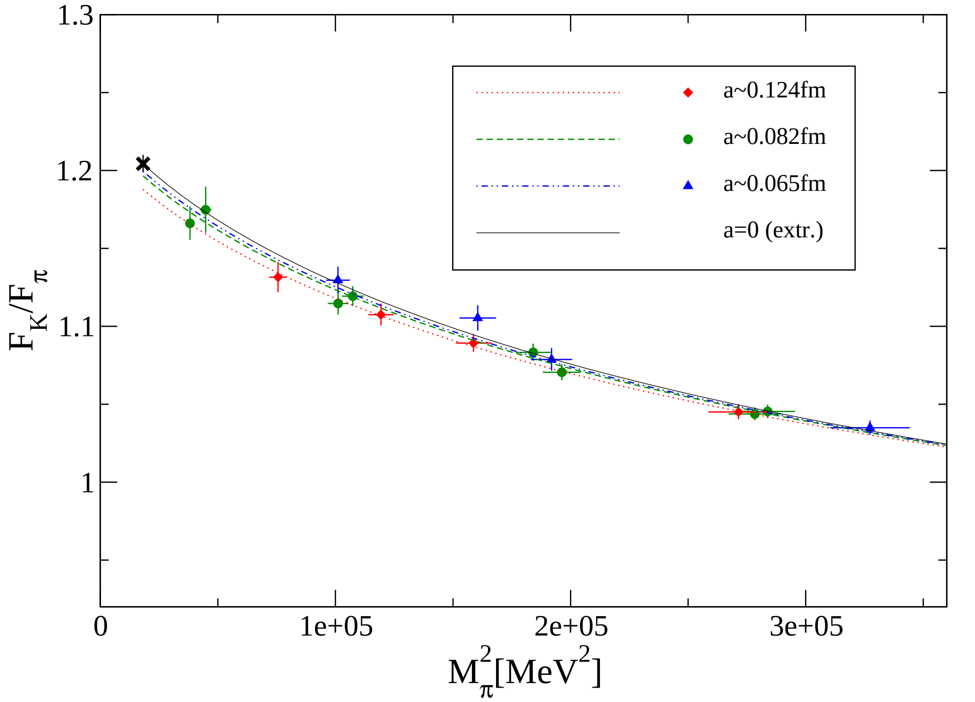
$$+\frac{4}{F_0^2}[M_K^2 - M_\pi^2]\left\{L_5(\mu) + P_{ud}M_\pi^2 + P_s[M_K^2 - M_\pi^2/2]\right\}$$

For instance, set scale w/ M_Ξ, fix $F_0 = F_\pi$ and include discretization errors through $F_0 \to F_0(1 + \alpha_F a^2)$

Excellent fit w/ $\mathrm{CL} = 31\%$ and

$$\frac{F_K}{F_\pi} = 1.204(5)(??)\ [0.4\%][??]$$

and aim to get ?? below 1%.



F_K/F_π from the lattice: comparison

Ref.	N_f	action	a/fm	Lm_π	m_π/MeV	F_K/F_π	$L_5 \cdot 10^3$
PDG '06						1.223(15)	
Bijnens '07 $O(p^4)$							1.46
Bijnens '07 $O(p^6)$							0.97(11)
ETM '07	2	tmQCD	$0.09[f_\pi]$	3.2	$\gtrsim 290$	1.227(9)(24)	
MILC '04-'07	2+1	$\mathrm{KS}^{\mathrm{AsqTad}}_{\mathrm{MILC}}$	$\gtrsim 0.06[f_\pi]$	4	$\gtrsim 240$	$1.197(3)(^{+6}_{-13})$	$1.4(2)(^{+2}_{-13})$
HPQCD-UKQCD '07	2+1	$\mathrm{KS}^{\mathrm{HISQ}}_{\mathrm{MILC}}$	$\gtrsim 0.09[\Upsilon]$	3.8	$\gtrsim 250$	1.189(7)	
NPLQCD '06	2+1	$\mathrm{KS}_{\mathrm{MILC}}$ /DWF	$0.13[r_0]$	3.7	$\gtrsim 290$	$1.218(2)(^{+11}_{-24})$	
RBC-UKQCD '07-'08	2+1	DWF	$0.11[\Omega]$	4.6	$\gtrsim 330$	1.205(18)	0.86(10)
PACS-CS '07	2+1	NP-SW	$0.09[\phi]$	3	$\gtrsim 210$	1.219(26)	1.47(13)
This work	2+1	SW	$\gtrsim 0.065[\Xi]$	> 4	$\gtrsim 190$	1.204(5)(??)	1.10(11)(??)

Conclusion

- Lattice QCD simulations have made tremendous progress in the last few years
- It is now possible to perform $2+1$ flavor lattice calculations near the physical QCD point ($M_\pi = 135\,\mathrm{MeV}$, $a \to 0$, $L \to \infty$)
- The light hadron spectrum, obtained w/ a $2+1$ flavor calculation in which extrapolations to the physical point are controlled, is in excellent agreement with the measured spectrum
- A calculation of F_K/F_π in the same approach should allow for a very competitive determination of $|V_{us}|$ as well as stringent tests of the SM and constraints on NP
- Many more quantities are being computed: quark masses, strange, charm and bottom weak matrix elements, etc.
- The age of precision non-perturbative QCD calculations is finally dawning