RI-MOM renormalization of twist-2 and bilinear quark operators with twisted mass fermions

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Outline

- Introduction
  - Why twist-2 and bilinear operators
  - RI-MOM renormalization scheme
- preliminary results
  - quark propagators and Z_q
  - Z_S, Z_P, Z_{44}
- summary and to do

Please see Gregorio Herdoiza's talk for tmQCD and ETMC.

Introduction

- Hadron structure functions contain information about the internal structure of hadrons. For example, The spin-averaged structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ can tell us overall densities of quarks and gluons in a hadron.
- Operator product expansion (OPE) can connect moments of structure functions with hadron matrix elements of local operators. For example, to leading twist order (twist-2)

$$2\int_0^1 x^{n-1}F_1(x, Q^2) = \sum_f C_{1,n}^{(f)}(\mu^2/Q^2)v_n^{(f)}(\mu),$$

where the reduced matrix element $v_n^{(f)}$ is defined by

$$\frac{1}{2}\sum_s \langle N(\vec{p}, s)|\mathcal{O}_f^{\{\mu_1\cdots\mu_n\}} - \text{traces}|N(\vec{p}, s)\rangle$$
$$= 2v_n^{(f)}(p^{\mu_1}\cdots p^{\mu_n} - \text{traces}). \qquad (1)$$

$\{\cdots\}$ means symmetrization on the Lorentz indices.

Introduction

- Here the twist-2 operator

$$\mathcal{O}_f^{\mu_1 \cdots \mu_n} = (\frac{i}{2})^{n-1} \bar{\psi}_f \gamma^{\mu_1} \overleftrightarrow{D}^{\mu_2} \cdots \overleftrightarrow{D}^{\mu_n} \psi_f ,$$

$\overleftrightarrow{D} = \overleftarrow{D} - \overrightarrow{D}$.

- These hadron matrix elements can be calculated on the lattice. We need the renormalization constants to match the bare results to other schemes.
- To calculate the lowest moment ($n = 2$), a very often used twist-2 operator is

$$\mathcal{O}_{44}(x) = \frac{1}{2} \bar{u}(x)[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3} \sum_{k=1}^{3} \gamma_k \overleftrightarrow{D}_k] u(x),$$

where $D_\mu = \frac{1}{2}(\bigtriangledown_\mu + \bigtriangledown_\mu^*)$.

Introduction

- To connect quark masses, the chiral condensate, nucleon axial coupling constant etc. calculated on the lattice to experiment results or other theory calculations , we need renormalization constants for quark bilinear operators $\bar{\psi}\Gamma\psi$, $\Gamma = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$.
- Lattice perturbation calculations of renormalization constants do not converge very well and rarely extend beyond the one-loop level.
- We are using a non-perturbative method, the RI-MOM scheme. When required, the connection to other schemes, e.g. $\overline{MS}$, can be computed with continuum perturbation theory.

The RI-MOM scheme

- The renormalization condition in the RI-MOM scheme is *[G. Martinelli et al., Nucl. Phys. B* **445** *(1995) 81]*

$$Z_q Z_{\mathcal{O}} \frac{1}{12} \mathrm{Tr}[\Lambda_{\mathcal{O}}(p) \Lambda_{\mathcal{O}}^{tree}(p)^{-1}]_{p^2=\mu^2} = 1,$$

  where Z_q is the quark field renormalization constant: $\psi_R = Z_q^{-1/2}\psi$, $Z_{\mathcal{O}}$ is the renormalization constant for the operator $\mathcal{O}$: $\mathcal{O}_R = Z_{\mathcal{O}}\mathcal{O}$, μ is the renormalization scale.
- $\Lambda_{\mathcal{O}}(p)$ is the amputated forward Green function

$$\Lambda_{\mathcal{O}}(p) = S^{-1}(p) G_{\mathcal{O}}(p) S^{-1}(p),$$

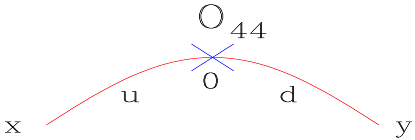
  where $S(p)$ is the quark propagator.
- The calculation has to be done in a fixed gauge, say, Landau gauge. The method is supposed to work when μ satisfies

$$\Lambda_{QCD} \ll \mu \ll \pi/a.$$

The RI-MOM scheme

- The forward Green's function $G_{\mathcal{O}}(p)$ is computed by

$$G_{\mathcal{O}}(p) = \sum_{x,y} e^{-ip\cdot(x-y)} \langle \psi(x)\mathcal{O}(0)\bar{\psi}(y)\rangle .$$



- For quark bilinears $\bar{u}\Gamma d$,

$$\begin{aligned} G_{\Gamma}(p) &= \sum_{x,y} e^{-ip\cdot(x-y)} \langle u(x)\bar{u}(0)\Gamma d(0)\bar{d}(y)\rangle \\ &= \frac{1}{N}\sum_{i=1}^{N} S_{u,i}(p|0)\Gamma\gamma_5 S^{\dagger}_{d,i}(p|0)\gamma_5, \qquad (2) \end{aligned}$$

where N is the total number of gauge configurations and

$$S_{u/d,i}(p|0) = \sum_{x} e^{-ip\cdot x} S_{u/d,i}(x,0).$$

The RI-MOM scheme

- The quark propagator in momentum space is given by

$$S_{u/d}(p) = \frac{1}{N}\sum_{i=1}^{N} S_{u/d,i}(p|0).$$

- At tree level, $\Lambda_\Gamma^{tree}(p) = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ for the quark bilinear $\bar{u}\Gamma d$.
- The quark field renormalization constant Z_q can be obtained by comparing the quark propagator to the free propagator (RI' scheme):

$$Z_q^{RI'} = \frac{-i}{12}\mathrm{Tr}\left[S(p)\gamma_\mu \sin(p_\mu a)\right]_{p^2=\mu^2}.$$

- In the RI scheme, Z_q is calculated from

$$(Z_q^{RI})^{-1} = \frac{i}{48}\mathrm{Tr}\left[\gamma_\mu \frac{\partial S^{-1}(p)}{\partial p_\mu}\right]_{p^2=\mu^2}.$$

- The difference between the two schemes is at the N^2LO in the Landau gauge. *[E. Franco and V. Lubicz 1998, K. G. Chetyrkin and A. Retey 2000]*

$$\frac{Z_q^{RI'}}{Z_q^{RI}} = 1 - \left(\frac{67}{6} - \frac{2n_f}{3}\right)\left(\frac{\alpha_s}{4\pi}\right)^2 - \left(\frac{52321}{72} - \frac{2236n_f}{27} + \frac{52n_f^2}{27} - \frac{607\zeta_3}{4} + 8\zeta_3 n_f\right)\left(\frac{\alpha_s}{4\pi}\right)^3 + O(\alpha_s^4).$$

- For twist-2 operators, we are considering

$$\mathcal{O}_{44}(x) = \frac{1}{2}\bar{u}(x)[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k]d(x),$$

where $D_\mu = \frac{1}{2}(\bigtriangledown_\mu + \bigtriangledown_\mu^*)$.

- Finishing the Wick contractions, we find

$$\langle u(x)\mathcal{O}_{44}(0)\bar{d}(y)\rangle = -\frac{1}{4}S_u(x,0)\gamma_5\tilde{S}_d^\dagger(y,0)\gamma_5 -\frac{1}{4}\tilde{S}_u(x,0)\gamma_5 S_d^\dagger(y,0)\gamma_5, \quad (3)$$

where $\tilde{S}_u(y,0)$ is defined by (similarly for $\tilde{S}_d$)

$$\tilde{S}_u(y,0) \equiv S_u(y,\hat{t})U_{\hat{t}}^\dagger(0)\gamma_4 - S_u(y,-\hat{t})U_{\hat{t}}(-\hat{t})\gamma_4 -\frac{1}{3}\left[S_u(y,\hat{k})U_{\hat{k}}^\dagger(0)\gamma_k - S_u(y,-\hat{k})U_{\hat{k}}(-\hat{k})\gamma_k\right],$$

and thus can be computed by solving

$$\sum_y D_u(z,y)\tilde{S}_u(y,0) = \delta_{z,\hat{t}}U_{\hat{t}}^\dagger(0)\gamma_4 - \delta_{z,-\hat{t}}U_{\hat{t}}(-\hat{t})\gamma_4 - \frac{1}{3}\left[\delta_{z,\hat{k}}U_{\hat{k}}^\dagger(0)\gamma_k - \delta_{z,-\hat{k}}U_{\hat{k}}(-\hat{k})\gamma_k\right].$$

Here $D_u(z,y)$ is the Dirac operator for the up quark.

- Then the Green function for $\mathcal{O}_{44}$ is

$$G_{\mathcal{O}_{44}}(p) = \sum_{x,y} e^{-ip\cdot(x-y)} \langle u(x) \mathcal{O}_{44}(0) \bar{d}(y) \rangle$$
$$= -\frac{1}{4} \cdot \frac{1}{N} \sum_{i=1}^{N} \Big[S_{u,i}(p|0) \gamma_5 \tilde{S}^{\dagger}_{d,i}(p|0) \gamma_5 + \tilde{S}_{u,i}(p|0) \gamma_5 S^{\dagger}_{d,i}(p|0) \gamma_5 \Big], \quad (4)$$

where N is the number of configurations and

$$\tilde{S}^{\dagger}_{u/d,i}(p|0) = \sum_{y} e^{ip\cdot y} \tilde{S}^{\dagger}_{u/d,i}(y,0).$$

- At tree level, we have

$$\Lambda^{tree}_{\mathcal{O}_{44}}(p) = i \left[\gamma_4 p_4 - \frac{1}{3} \sum_{k=1}^{3} \gamma_k p_k \right].$$

- To obtain the quark propagators in the Landau gauge, we can fix the gauge and then invert. For $a\mu_q = 0.0064$ on the $24^3 \times 48$ lattice, one propagator (12 inversions) takes ~ 26 hours on a Xeon5150 2.66GHz machine.
- Since the point source quark propagators have been calculated for our baryon spectrum project without gauge fixing, we can fix the gauge and then convert the propagators. ~ 45 minutes for one propagator.
- $$U^f_\mu(x) = G(x) U^i_\mu(x) G^\dagger(x+\hat{\mu}), \quad G(x) \in SU(3).$$
$$U_p(x) \equiv U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x),$$
$$U^f_p(x) = G(x) U^i_p(x) G^\dagger(x).$$
- $$S^f(x,y;U^f) = G(x) S^i(x,y;U^i) G^\dagger(y).$$

Preliminary results

- Use a unit gauge configuration to check the analysis codes.
- Dynamical simulations of $N_f = 2$ degenerate Wilson twisted mass quarks. Tree-level Symanzik improved gauge action.
- $\beta = 3.9$, $a\mu_q = 0.0064, 0.0085$, $24^3 \times 48$, 30 configs per mass.
- The momenta take the values

$$ap_\mu = \left(\frac{\pi}{T}(2k_t + 1), \frac{2\pi}{L}k_x, \frac{2\pi}{L}k_y, \frac{2\pi}{L}k_z \right), \tag{5}$$

  where (k_t, k_x, k_y, k_z)=(0,0,0,0),...,(3,4,4,4). [$ap < \pi/2$]
- We are using the hypercubic improved method described in [F. de Soto and C. Roiesnel, JHEP **0709** (2007) 007] to average (k_t, k_x, k_y, k_z)'s corresponding to a same p^2 to reduce hypercubic lattice artifacts.
- We also use the "democratic" method (average data points close to the diagonal line) to compare. The errors are from Jackknife.

quark propagators, $Z_q^{RI'}$

- The inverse of the full quark propagator in the twisted basis takes the form

$$S_{tw}^{-1}(p) = -i\gamma_\mu \sin(p_\mu a)\Sigma_1(p^2) - \Sigma_3(p^2) \pm i\gamma_5\Sigma_2(p^2).$$

$\pm$: up and down. At tree-level, $\Sigma_1(p^2) = 1$, $\Sigma_2(p^2) = a\mu_q$ and $\Sigma_3(p^2) = 2\sum_{\mu=1}^{4} \sin^2(\frac{p_\mu a}{2})$.

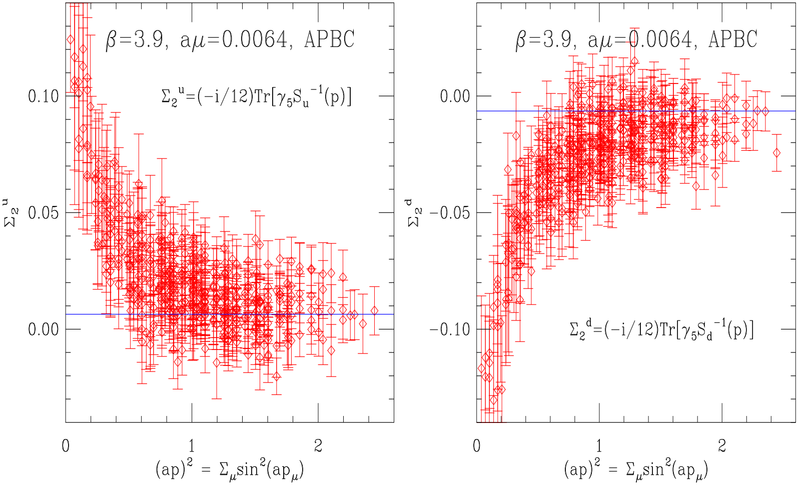
- $$\Sigma_1(p^2) = \frac{i}{12a^2p^2}\mathrm{Tr}[\gamma_\mu \sin(p_\mu a) S_{tw}^{-1}(p)]$$

- $$\Sigma_2(p^2) = \mp\frac{i}{12}\mathrm{Tr}[\gamma_5 S_{tw}^{-1}(p)]$$

- $$\Sigma_3(p^2) = \frac{-1}{12}\mathrm{Tr}[S_{tw}^{-1}(p)]$$

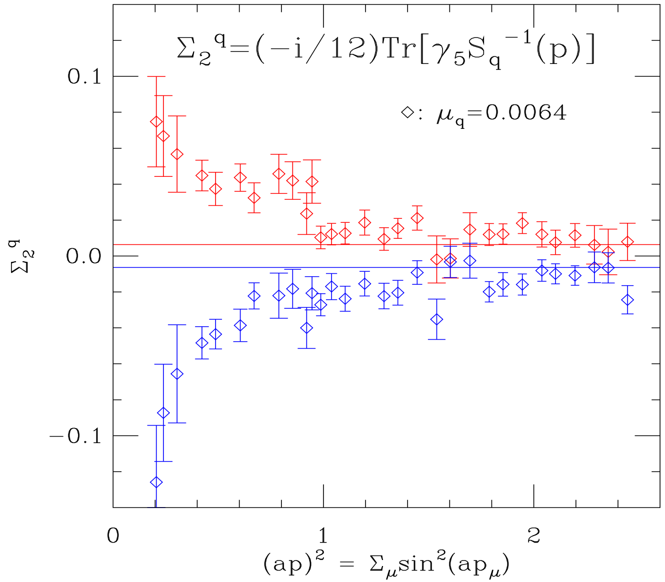
- $$Z_q^{RI'}(\mu^2) = (\Sigma_1(\mu^2))^{-1}$$

Σ_2

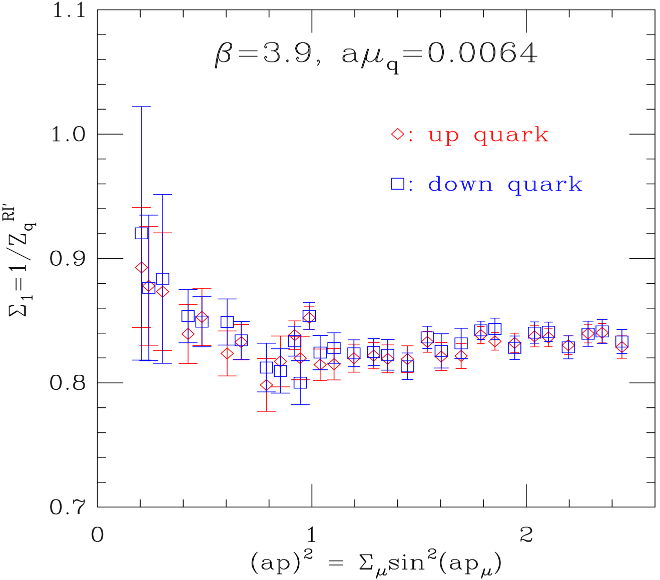


β=3.9, aμ=0.0064, APBC
Σ2u=(−i/12)Tr[γ5Su−1(p)]
Σ2u
0.10
0.05
0.00
0
1
2
(ap)2 = Σμsin2(apμ)
β=3.9, aμ=0.0064, APBC
Σ2d=(−i/12)Tr[γ5Sd−1(p)]
Σ2d
0.00
−0.05
−0.10
0
1
2
(ap)2 = Σμsin2(apμ)

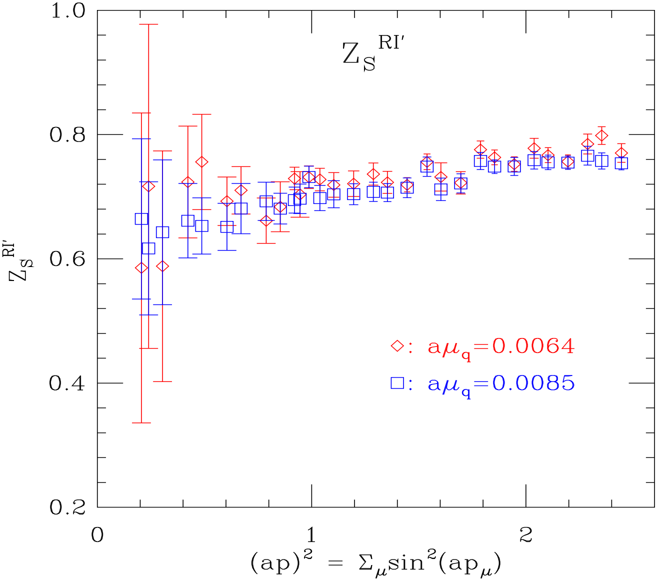
Σ_2, averaging (k_t, k_x, k_y, k_z)'s close to the diagonal



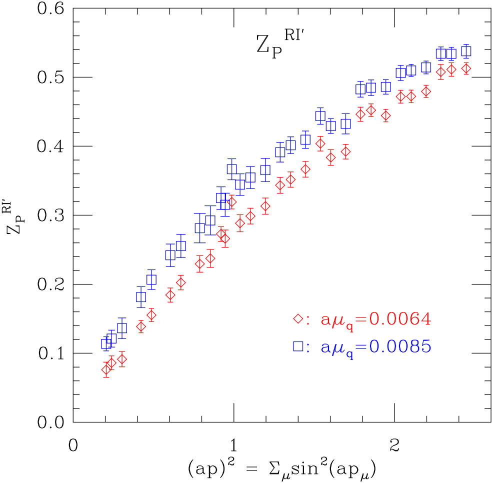
$\Sigma_1(= 1/Z_q^{RI'})$



Z_S for the scalar density $\bar{u}d$

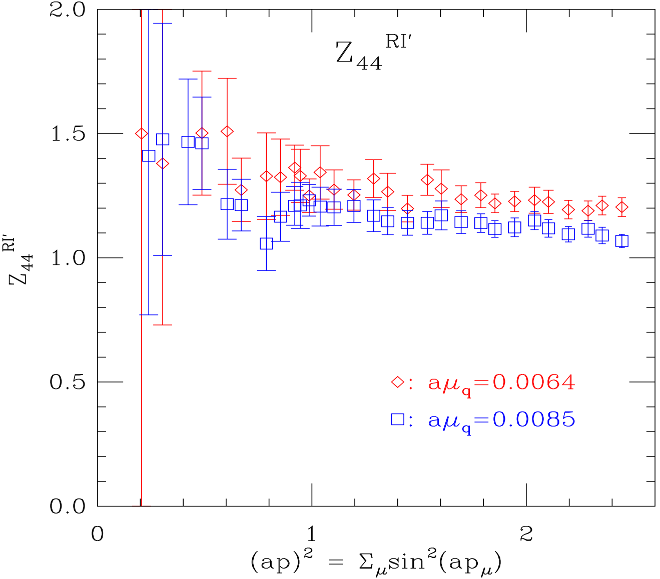


Z_P for the pseudoscalar density $\bar{u}\gamma_5 d$

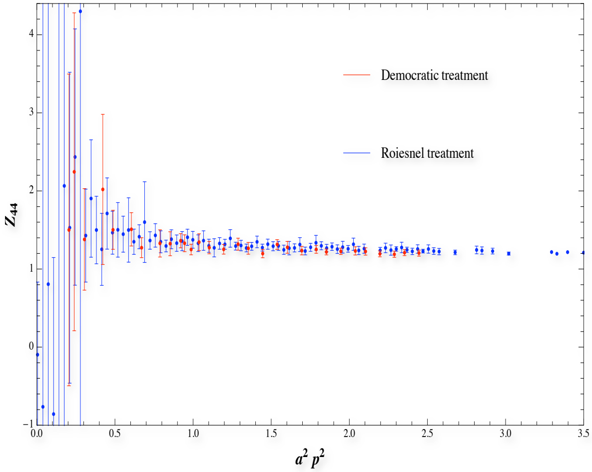


$$\Gamma_P(p^2, m) \equiv \frac{1}{12}\mathrm{Tr}[\Lambda_P \gamma_5] = c_1(p^2, m) + c_2(p^2, m)\frac{\langle \bar{q}q \rangle}{mp^2} + \mathcal{O}(\frac{1}{p^4}),$$

Z_{44} for operator $\mathcal{O}_{44}$



Hypercubic improved vs. democratic average: Z_{44}, $a\mu = 0.0064$



Summary and to do

- We have obtained some preliminary results for Z_{44}, Z_S and Z_P using the RI-MOM scheme for two flavor twisted mass fermion simulations.
- The statistics are low. We are calculating with more configurations (up to $\sim$ 300 for each mass), quark masses and beta values.
- Extrapolate to the chiral limit. Subtract the Goldstone pole for Z_P.
- Conversion to the $\overline{MS}$ scheme.
- More twist-2 operators, Z_V, Z_A etc.

Thanks for your attention !