

# THE PION-NUCLEON SIGMA TERM: an introduction

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Réunion du GDR *Physique subatomique et calculs sur réseau*  
CPT Luminy, June 25-27, 2008

# OUTLINE

- Introduction
- Dispersive determinations of the sigma term
- Summary

# Introduction (1)

## Definition

$$\sigma_N \equiv \frac{1}{2m_N} \langle N(p) | \widehat{m} (\bar{u}u + \bar{d}d) (0) | N(p) \rangle$$

$$m_N = \text{nucleon mass}, \quad \widehat{m} \equiv (m_u + m_d)/2$$

$\sigma_N$  is related to

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- The  $\pi - N$  scattering amplitude
- The detection of dark matter see e.g. [Ellis et al., Phys. Rev. D 77, 065026 (2008)]

PHYSICAL REVIEW D 77, 065026 (2008)

## Hadronic uncertainties in the elastic scattering of supersymmetric dark matter

John Ellis,<sup>1,\*</sup> Keith A. Olive,<sup>2,†</sup> and Christopher Savage<sup>2,‡</sup>

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(Received 28 January 2008; published 27 March 2008)

We review the uncertainties in the spin-independent and spin-dependent elastic scattering cross sections of supersymmetric dark matter particles on protons and neutrons. We propagate the uncertainties in quark masses and hadronic matrix elements that are related to the  $\pi$ -nucleon  $\sigma$  term and the spin content of the nucleon. By far the largest single uncertainty is that in spin-independent scattering induced by our ignorance of the  $\langle N|qq|N\rangle$  matrix elements linked to the  $\pi$ -nucleon  $\sigma$  term, which affects the ratio of cross sections on proton and neutron targets as well as their absolute values. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. We plead for an experimental campaign to determine better the  $\pi$ -nucleon  $\sigma$  term. Uncertainties in the spin content of the proton affect significantly, but less strongly, the calculation of rates used in indirect searches.



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- The quark mass ratio
- The  $\pi - N$  scattering amplitude
- The detection of dark matter
- ...

# Introduction (2)

## The sigma term and the hadron mass spectrum

Through the Feynman - Hellmann theorem

$$\frac{\partial E(\lambda)}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial H(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle$$

$$H(\lambda) | \psi(\lambda) \rangle = E(\lambda) | \psi(\lambda) \rangle, \quad \langle \psi(\lambda) | \psi(\lambda) \rangle = 1$$

adapted to QFT [L. S. Brown, Phys. Rev. 187, 2260 (1969)]

$$\frac{\partial m_h^2(m_q)}{\partial m_q} = \langle h(p) | \frac{\partial \mathcal{H}^{QCD}(0)}{\partial m_q} | h(p) \rangle$$

$$\begin{aligned} \mathcal{H}^{QCD}(0) = -\mathcal{L}^{QCD}(0) &= \dots + m_u \bar{u}u(0) + m_d \bar{d}d(0) + m_s \bar{s}s(0) \\ &= \dots + \hat{m} (\bar{u}u + \bar{d}d)(0) + \frac{m_u - m_d}{2} (\bar{u}u - \bar{d}d)(0) + m_s \bar{s}s(0) \end{aligned}$$

# Introduction (2)

## The sigma term and the hadron mass spectrum

- example : the pion

$$\begin{aligned}\frac{\partial M_\pi^2}{\partial \widehat{m}} &= \langle \pi(p) | (\bar{u}u + \bar{d}d) (0) | \pi(p) \rangle, & \frac{\partial M_\pi^2}{\partial m_s} &= \langle \pi(p) | \bar{s}s(0) | \pi(p) \rangle \\ \rightarrow \frac{\partial M_\pi^2}{\partial \widehat{m}} &= -\frac{\langle \bar{q}q \rangle_0}{F_0^2} + \mathcal{O}(m_u, m_d, m_s), & \frac{\partial M_\pi^2}{\partial m_s} &= 0 + \mathcal{O}(m_u, m_d, m_s)\end{aligned}$$

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- example : the nucleon

$$\begin{aligned}\frac{\partial m_N^2}{\partial \widehat{m}} &= \langle N(p) | (\bar{u}u + \bar{d}d) (0) | N(p) \rangle \\ \rightarrow m_N &= \overset{o}{m}_N + \sigma_N + \dots\end{aligned}$$

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quark masses cannot be varied in the real world...

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15 – 25 MeV [S. Cabasino et al., Nucl. Phys. B (Proc. Suppl.) 20, 399 (1991)]

40 – 60 MeV [Fukugita et al., Phys. Rev. D 51, 5319 (1995)]

$50 \pm 3$  MeV [Dong et al., Phys. Rev. D 54, 5496 (1996)]

45 – 55 MeV [Leinweber et al., Phys. Lett. B 482, 109 (2000)]

$18 \pm 5$  MeV [Gusten et al., Phys. Rev. D 59, 054504 (1999)]

$49 \pm 3$  MeV [Procuero et al., Phys. Rev. D 69, 034505 (2004)]

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- difficult to believe that some of the error bars are reliable
- not straightforward to extract an estimate of  $\sigma_N$



# Introduction (3)

The sigma term, the quark mass ratio  $m_s/\widehat{m}$ , and the strangeness content of the nucleon  $y$

$$\left(\frac{m_s}{\widehat{m}} - 1\right) (1 - y)\sigma_N = \frac{m_s - \widehat{m}}{2m_N} \langle N(p) | (\bar{u}u + \bar{d}d - 2\bar{s}s) (0) | N(p) \rangle$$

$$y \equiv 2 \frac{\langle N(p) | (\bar{s}s) (0) | N(p) \rangle}{\langle N(p) | (\bar{u}u + \bar{d}d) (0) | N(p) \rangle}$$

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At lowest order [Gasser + Leutwyler, Phys. Rep. 87, 77 (1982)],

$$\left(\frac{m_s}{\widehat{m}} - 1\right) (1 - y)\sigma_N \sim m_\Xi + m_\Sigma - 2m_N \sim 634 \text{ MeV} \quad [\mathcal{O}(m_q)]$$

from  $SU(3)$  breaking in the masses of the baryon octet.

Theoretical prejudice

$$m_s/\widehat{m} \sim 25 [M_K^2/M_\pi^2, \text{lattice}], \quad y \sim 0.2 [\text{Zweig rule, large } N_C]$$

$$\rightarrow \sigma_N \sim 33 \text{ MeV}$$

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Higher order corrections,

$$\begin{aligned} (1 - y)\sigma_N &= 26 \text{ MeV } \mathcal{O}(m_q) \\ &= (35 \pm 5) \text{ MeV } \mathcal{O}(m_q^{3/2}) \text{ [Gasser, Ann. Phys. (N. Y.) 136, 62, (1981)]} \\ &= (36 \pm 7) \text{ MeV } \mathcal{O}(m_q^2) \text{ [Borasoy + Meißner, Ann. Phys. (N. Y.) 254, 192, (1997)]} \\ &\rightarrow \sigma_N \sim 45 \text{ MeV} \quad [y \sim 0.2] \end{aligned}$$

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$$\rightarrow \sigma_N \sim 45 \text{ MeV} \quad [y \sim 0.2]$$

increasing  $\sigma_N$  means reducing  $m_s/\widehat{m}$  and/or increasing  $y$  !!!

# Introduction (4)

## The sigma term and the $\pi - N$ scattering amplitude

A low-energy theorem [Cheng + Dashen, Phys. Rev. Lett. 26, 574 (1971)] states that

$$\Sigma = \sigma_N \left[ 1 + \mathcal{O}(m_q^{1/2}) \right]$$

where

$$\Sigma \equiv F_\pi^2 \overline{D}^+(\nu = 0, t = 2M_\pi^2)$$

is defined in terms of the  $\pi - N$  scattering amplitude  $\pi(q)N(p) \rightarrow \pi(q')N(p')$

$$T_{\pi N}(\nu, t) = \bar{u}(p') \left[ A(\nu, t) + \frac{1}{2} \gamma^\mu (q + q')_\mu B(\nu, t) \right] u(p)$$

$$t = (p - p')^2, \nu = (s - u)/4m_N$$

[Höhler, Pion-Nucleon Scattering, Landolt-Börnstein vol. I/9b]

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$$\Sigma \equiv F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2)$$

$$D(\nu, t) = A(\nu, t) + \nu B(\nu, t), \quad D^\pm(\nu, t) = \frac{1}{2} [D_{\pi-p}(\nu, t) \pm D_{\pi+p}(\nu, t)]$$

$$\bar{D}^+(\nu = 0, t) = D^+(\nu = 0, t) - \frac{g_{\pi N}^2}{m_N}$$

Drawback:  $\Sigma/\sigma_N - 1 = \mathcal{O}(m_q^{1/2})$ , large chiral corrections expected

# Introduction (4)

## The sigma term and the $\pi - N$ scattering amplitude

Improved low-energy theorem [Brown, Pardee, Peccei, Phys. Rev. D 4, 2901 (1971)]

The sigma term can be viewed as the scalar form factor of the nucleon at zero momentum transfer

$$\sigma_N \equiv \sigma_N(t = 0), \quad \sigma_N(t) \equiv \frac{1}{2m_N} \langle N(p') | \widehat{m} (\bar{u}u + \bar{d}d) (0) | N(p) \rangle$$

$$t = (p' - p)^2$$

Then

$$\Sigma = \sigma_N(2M_\pi^2) + \Delta_R, \quad \Delta_R = \mathcal{O}(m_q^2)$$

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$$\Sigma = \sigma_N(2M_\pi^2) + \Delta_R, \quad \Delta_R = \mathcal{O}(m_q^2)$$

$\Delta_R$  can be evaluated in ChPT

$$\Delta_R = 0.35 \text{ MeV} \quad [\text{Gasser, Sainio, Švarc, Nucl. Phys. B 307, 779 (1988)}]$$

$$\sim 2 \text{ MeV} \quad [\text{Bernard, Kaiser, Meißner, Phys. Lett. B 389, 144 (1996)}]$$



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The sigma term and the  $\pi - N$  scattering amplitude

There remains to fix  $\Delta_\sigma \equiv \sigma_N(2M_\pi^2) - \sigma_N(0)$

$$\Delta_\sigma \sim 5 \text{ MeV} \quad [\text{Gasser, Sainio, Švarc, Nucl. Phys. B 307, 779 (1988)}]$$

$$= 15.2 \pm 0.4 \text{ MeV} \quad [\text{Gasser, Leutwyler, Sainio, Phys. Lett. B 253, 260 (1991)}]$$

$$= 14.0 \text{ MeV} + 2M_\pi^4 \bar{e}_2 \quad [\text{Becher, Leutwyler, JHEP 6, 017 (2001)}]$$

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Conclusion

$$\sigma_N = \Sigma - \Delta_\sigma - \Delta_R, \quad \Delta_\sigma + \Delta_R \sim 16 \text{ MeV}$$

$$\Sigma ?$$

# Dispersive determinations of the sigma term

The CD point is outside of the physical region for  $\pi - N$  scattering where data exist

Unitarity and analyticity properties allow to obtain information on the amplitude inside the Mandelstam triangle from data in the physical region  $\longrightarrow$  dispersion relations

This is a complex and involved machinery (hyperbolic dispersion relations, subtraction constants, partial wave analysis of  $\pi - N$  scattering data,...)

This program was carried through by the Karlsruhe group at the beginning of the eighties

$$\Sigma = 64 \pm 8 \text{ MeV}$$

[Koch, Z. Phys. C 15, 161 (1982)]

# Dispersive determinations of the sigma term

Inside the Mandelstam triangle, the amplitude  $\bar{D}^+(\nu, t)$  is a real and smooth function

Near the center of the Mandelstam triangle, the amplitude  $\bar{D}^+(\nu, t)$  can be represented by a **subthreshold expansion**

$$\bar{D}^+(\nu, t) = \sum_{m,n} d_{mn}^+ \nu^{2m} t^n$$

$$\bar{D}^+(0, 2M_\pi^2) = d_{00}^+ + 2M_\pi^2 d_{01}^+ + 4M_\pi^4 d_{02}^+ + \dots$$

$$d_{00}^+ = (-1.46 \pm 0.10) M_\pi^{-1}, \quad d_{01}^+ = (1.14 \pm 0.02) M_\pi^{-3}$$

[Höhler, Pion-Nucleon Scattering, Landolt-Börnstein vol. I/9b]

# Dispersive determinations of the sigma term

Since then, the situation has been in constant evolution

- New  $\pi - N$  data
- New partial wave analyses [Arndt et al., Stahov et al., Sainio et al...]
- More accurate information on  $\pi - \pi$  scattering

$$\Sigma = F_\pi^2 [d_{00}^+ + 2M_\pi^2 d_{01}^+] + \Delta_D$$

$$\Sigma = 59 \pm 2 \text{ MeV}, \quad \Delta_D \sim 12 \text{ MeV}$$

[Gasser et al., Phys. Lett. B 213, 85 (1988)]

... but some later analyses [VPI/GWU] claimed to obtain higher values

# Dispersive determinations of the sigma term

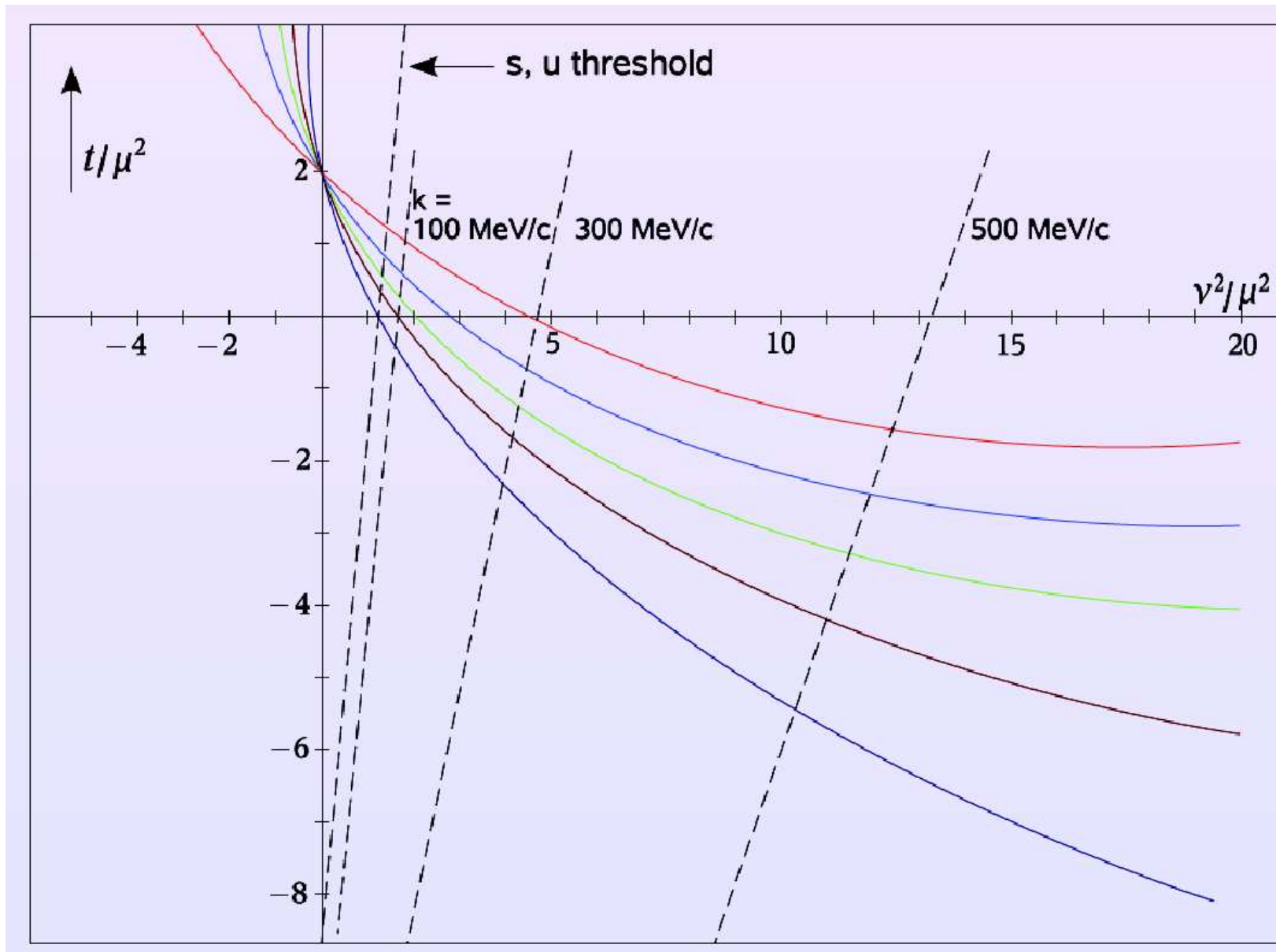
Two strategies can be developed:

- continuation of  $\overline{D}^+$  towards the CD point using hyperbolic dispersion relations

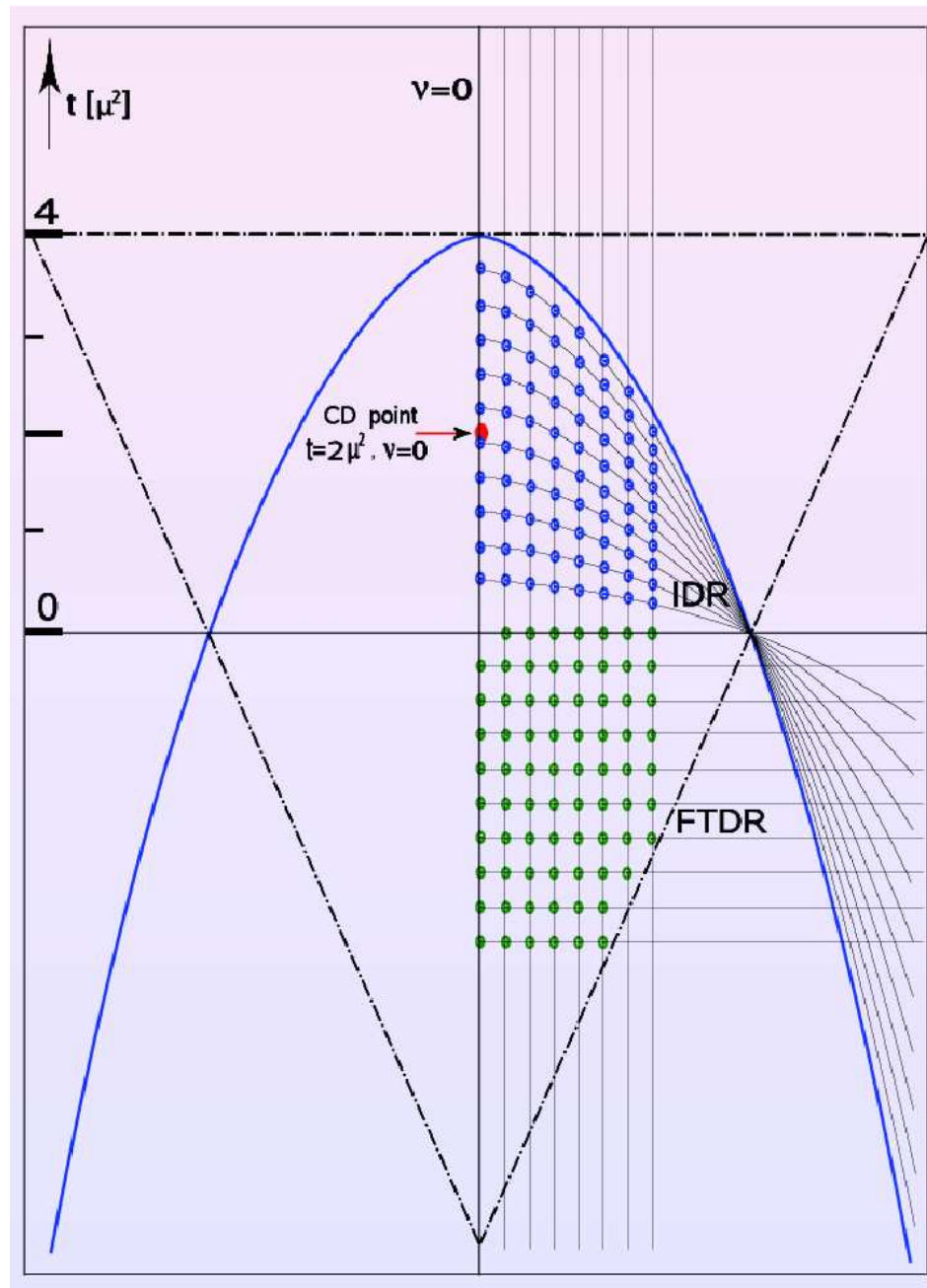
$$(\nu^2 - \nu_0^2)(t - t_0) = \frac{a^2}{2}, \quad a^2 = 2(t_0 - 2M_\pi^2)\nu_0^2$$

- determination of the subthreshold coefficients using both fixed- $t$  and interior dispersion relations

$$(s - a)(u - a) = b, \quad a < 0, \quad b = (m_N^2 - a - M_\pi^2) - 4aM_\pi^2$$



[Hadžimehmedović, Osmanović, Stahov, MENU 2007]





# Dispersive determinations of the sigma term

Most recent results along these lines

[Hadžimehmedović, Osmanović, Stahov, MENU 2007]

$$\overline{D}^+(0, 2M_\pi^2) = (1.185 \pm 0.033)M_\pi^{-1}$$

$$\Sigma = 72 \pm 2 \text{ MeV}$$

$$\begin{aligned} d_{00}^+ &= (-1.377 \pm 0.01)M_\pi^{-1} & d_{01}^+ &= (1.176 \pm 0.01)M_\pi^{-3} \\ d_{02}^+ &= (0.039 \pm 0.001)M_\pi^{-5} & d_{03}^+ &= (0.004 \pm 0.002)M_\pi^{-7} \end{aligned}$$

corresponding to

$$\Sigma_d = F_\pi^2 [d_{00}^+ + 2M_\pi^2 d_{01}^+] = 59.64 \text{ MeV}$$

$$\Delta_D = F_\pi^2 [2M_\pi^4 d_{02}^+ + 8M_\pi^6 d_{03}^+ + \dots] = 11.50 \text{ MeV} + \dots$$

$$\Sigma = 71 \pm 2 \text{ MeV}$$

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- ... are in disagreement with the result  $\Sigma = (81 \pm 6) \text{ MeV}$  [Hite et al., Phys. Rev. C 71, 065201 (2005)], using IDR along a single hyperbola going through the CD point, and essentially the same data

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- In view of the recent progress (dynamical quarks with chiral symmetry, low quark masses, continuum and infinite volume extrapolations), numerical simulations should become competitive
- Other related topics not covered in this introductions: hadronic atoms ( $\pi - N$  scattering lengths), sum rules (GMO, Olesen), **isospin breaking**