Nucleon axial coupling constant with Nf = 2 twisted mass fermions and other 3-point functions

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27 June 2008

Outline

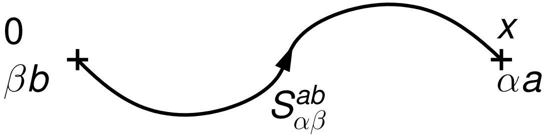
Propagator

- Key element obtained after wick contraction

$$S^{ab}_{\alpha\beta}(x,0) \;=\; \langle u^a_\alpha(x)\bar{u}^b_\beta(0)\rangle = \frac{1}{Z}\int D\bar{\psi}D\psi D\phi\, u(x)\bar{u}(0)e^{-S[\bar{\psi},\psi,\phi]}$$

- It obeys for a local source to

$$D[U]^{ca}_{\gamma\alpha}(z,x)S^{ab}_{\alpha\beta}(x,0) = \delta^{cb}\delta_{\gamma\beta}\delta(z,0)$$



- The source is fixed !
- Smearing

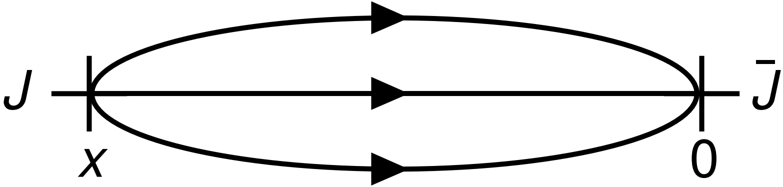
Contractions

- Interpolating field for nucleon

$$J_\alpha(x) = \epsilon^{abc} u^a_\alpha(\vec{x},t)\left[u^b(\vec{x},t)^{T_D} C\gamma_5 d^c(\vec{x},t)\right]$$

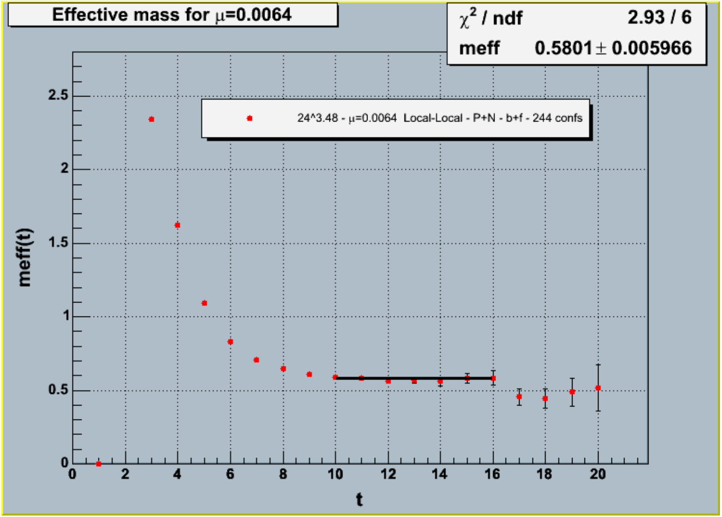
- Extract number from behaviour of correlator at large time

$$C_{\gamma\rho}(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \left\langle 0|J_\gamma(t,\vec{x})\bar{J}_\rho(0)|0\right\rangle \propto e^{-E(\vec{p})t}$$



Example

- Take $\vec{p} = 0$ and the correct combination of γ and ρ to get masses



Outline

Why ?

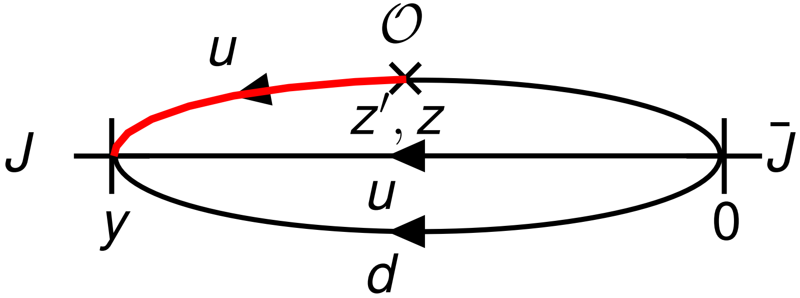
- g_A (Reminder : chiral extrapolation for nucleon $m_N = M_0 + c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 + \ldots$)
- Sigma term
- Moments of parton distribution functions (PDF)
- Moments of generalized parton distribution functions (GPDs)
- Form factors

Large experimental programs to study GPDs : Jlab, DESY(HERMES) and CERN(COMPASS)

How to compute

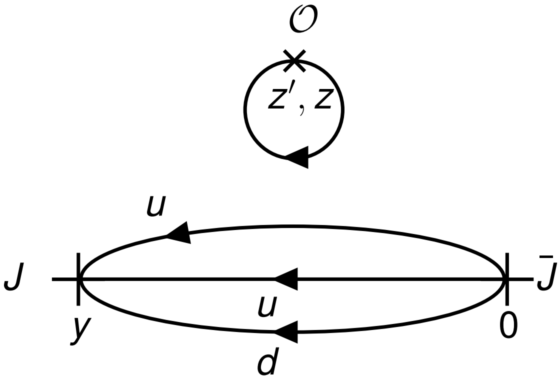
The matrix element of interest is now

$$
\begin{aligned}
R &= \sum_{\vec{y},\vec{z}} e^{i\vec{p}\cdot\vec{y}} e^{i\vec{p'}\cdot\vec{z}} \langle 0|J_\gamma(y)O(z)\bar{J}_\rho(0)|0\rangle \\
&\propto \langle N(p,s)|O(z)|N(p',s')\rangle \text{ if J is a nucleon interpolating field}
\end{aligned}
$$



- Try to look for a plateau on t_z with $t_y \gg t_z \gg 0$

Quark disconnected diagrams



- Evaluation of disconnected diagram numerically demanding (all to all propagator)
- Can look at non-singlet (u-d) matrix elements only

Generalized source

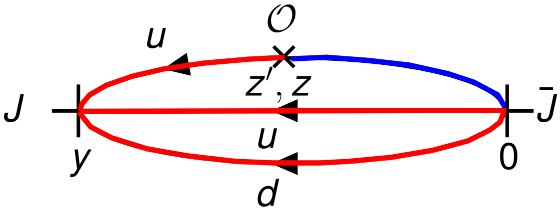
$$R = \sum_{\vec{y},\vec{z}} e^{i\vec{p}\cdot\vec{y}} e^{i\vec{p'}\cdot\vec{z}} \langle 0|J_\gamma(y) O(z) \bar{J}_\rho(0)|0\rangle$$

- After wick contractions obtain something like

$$R = B^\dagger \gamma_5 \Lambda S_u$$

- There is an unknown piece in B, but $B = D^{-1}\Sigma_G$ where Σ_G is a combination of "normal" propagators
- Σ_G is called generalized or sequential source
- New inversions are necessary
- To create Σ_G
  - Fix time of sink (ie t_y)
  - Fix momentum at sink but not where the operator is inserted
  - Choose how to project

Contractions



- Red part : generalized propagator
- Blue part : normal propagator

Just need to combine correctly Dirac, color, and space indices.

Renormalization issues

- We will use the RI-MOM scheme to renormalize non-perturbatively the bare quantities obtained (Zhaofeng's talk)
- Ideally (no mixing) multiplicative renormalization using scheme $\mathcal{S}$ at scale μ

$$\mathcal{O}^{\mathcal{S}}(\mu) = Z_{\mathcal{S}}^{\mathcal{O}}(\mu)\mathcal{O}_{bare}$$

- Some renormalization constants have already been computed by other members of the collaboration (Z_A, Z_V, ...)
- Perturbative theory can then be used to go to $\overline{\text{MS}}$ scheme

g_A

- Gives information about the fraction of spin carried by quarks
- Well known experimentally from neutron beta decay ($g_A \simeq 1.269$)
- Can be extracted with $O(z) = A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$
- We will need to use chiral perturbation theory, to check for finite size effects, for cutoff effects

$< x >$

- Gives information about the fraction of energy carried by quarks
- Twist-2 operators

$$\mathcal{O}^{\{\mu_1\cdots\mu_n\}} = (\frac{i}{2})^{n-1} G_{ff'} \bar{\psi}_f \gamma^{\{\mu_1} \overset{\leftrightarrow}{D}^{\mu_2} \cdots \overset{\leftrightarrow}{D}^{\mu_n\}} \psi_{f'} - traces,$$

$\overset{\leftrightarrow}{D} = \overset{\leftarrow}{D} - \vec{D}$, $\{\cdots\}$ means symmetrization on the Lorentz indices.

- In our calculation, we use the operator

$$\mathcal{O}_{44}(x) = \frac{1}{2}\bar{u}(x)[\gamma_4 \overset{\leftrightarrow}{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overset{\leftrightarrow}{D}_k]u(x),$$

where $D_\mu = \frac{1}{2}(\bigtriangledown_\mu + \bigtriangledown^*_\mu)$.

$< x >$

- Then the bare lowest moment of the quark distribution functions $\langle x \rangle$ is given by ($\vec{p} = 0$)

$$\langle x \rangle = \frac{1}{2m_N^2} \langle N, \vec{0} | \mathcal{O}_{44} | N, \vec{0} \rangle = \frac{2}{m_N} \frac{C_{44}(t)}{C_N(t_y)} \quad (0 \ll t \ll t_y).$$

Here

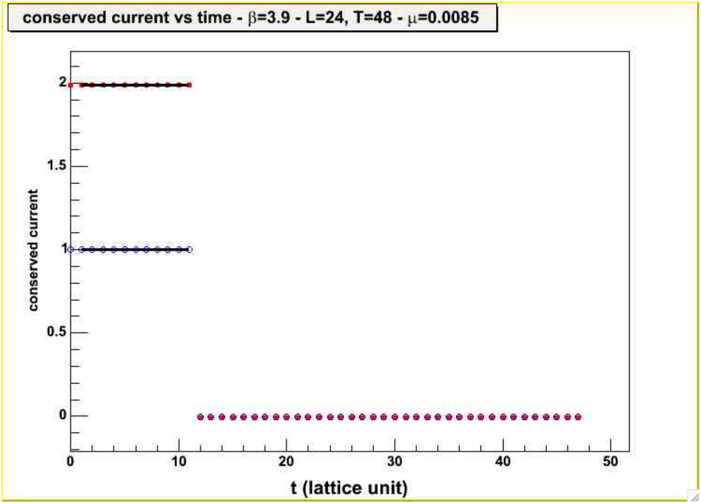
$$C_{44}(t) = \sum_{\vec{y},\vec{z}} \langle J(t_y, \vec{y}) \mathcal{O}_{44}(t, \vec{z}) \bar{J}(0, 0) \rangle,$$

$$C_N(t_y) = \sum_{\vec{y}} \langle J(t_y, \vec{y}) \bar{J}(0, 0) \rangle,$$

and $J(x)$ is the interpolating field for the nucleon.

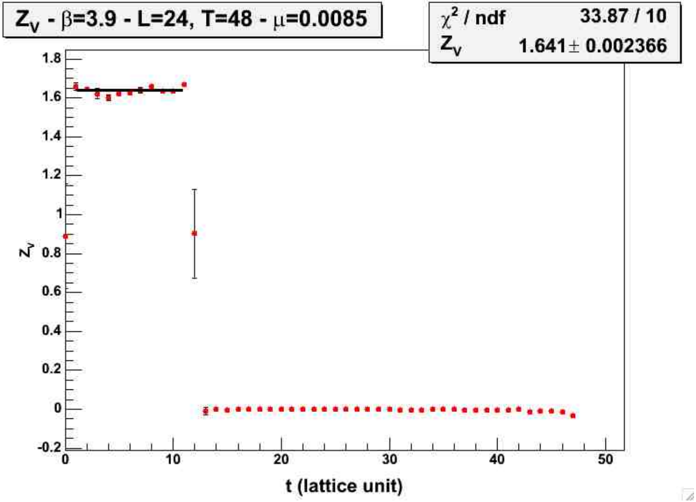
Plot

- Conserved (or point split) current (10 confs)



Plot

- Very preliminary (10 configurations)



- Close to the value found by italian members of the collaboration

Conclusion

- Wait for more data
- Future work to do
  - Higher moments of GPDs
  - Form factor at low transfer
  - Disconnected diagrams