Applications of Chiral Perturbation theory to lattice QCD (I)

Adapted and extended from [hep-lat/0607016]

Stephen R. Sharpe

University of Washington

Glossary!

- EFT Effective Field Theory
- χPT: Chiral Perturbation Theory
- PQQCD: Partially Quenched QCD
- PQχPT: Partially Quenched χPT
- WχPT: Wilson χPT (including lattice spacing effects)
- tmQCD: Twisted mass QCD
- tmχPT: Twisted mass χPT (including lattice spacing effects)
- SχPT: Staggered χPT (including lattice spacing effects)
- (P)GB: (Psuedo) Goldstone Boson
- LEC: Low energy coefficient (in chiral Lagrangian)
- VEV: Vacuum Expectation Value
- LO: leading order
- NLO: next-to-leading order, etc.
- NP: non-perturbative

Introduction and Overview

- Lattice QCD is at the beginning of an exciting era
  - Terascale ($\longrightarrow$ Petascale) computers
  - Unquenched simulations with $m_\pi \to 250$ MeV and below
  - Potential for few percent control over all systematics
- Yet LQCD simulations require extrapolations
  - To physical light quark masses, m_u, m_d
    - from $m_\ell = (m_u + m_d)/2 \sim (2-3) \times m_{\ell,\text{phys}}$
  - To the continuum limit, $a = 0$
    - from $a \sim 0.05 - 0.1$ fm, and from $\alpha_S \sim 0.3$
  - To "infinite" box size $L \gg 1/m_\pi$
    - from $L \sim 3 - 5$ fm
    - can also work directly in $\epsilon-$regime ($m_\pi L \lesssim 1$)
  - From $\alpha_{\text{EM}} = 0$ to $\alpha_{\text{EM}} = 1/137$
  - From finite volume energies of two particles to infinite volume scattering amplitudes
- **Theoretical input essential for these extrapolations!**

Widespread use of Unphysical Simulations

- Staggered fermions with the "$\sqrt[4]{\text{Det}}$" trick
  - Theory unitary (at best) in continuum limit
- "Mixed actions"
  - e.g. Overlap valence fermions on Wilson/tm sea
- Partially quenched QCD
  - Valence quarks *not degenerate* with sea quarks
  - Gives more information to constrain chiral extrapolations
- Matrix elements with unphysical kinematics
  - e.g. $\langle K|\mathcal{O}_W|\pi\pi\rangle$ with all particles at rest
  - $\mathcal{O}_W$ inserts momentum

All require theoretical input to obtain physical results

χPT is the tool for most extrapolations

Chiral Perturbation Theory for LQCD

Provides:

- Forms for chiral and $L \to \infty$ extrapolations
- Incorporation of operators with momentum insertion ($\mathcal{O}_W$)
- Extension to partially quenched theories: PQχPT
- Incorporation of lattice artifacts, particularly those breaking continuum symmetries
  - Wilson fermions (axial symmetry breaking): WχPT
  - Twisted mass (flavor symmetry breaking): tmχPT
  - Staggered fermions (taste symmetry breaking, $\sqrt[4]{\text{Det}}$ trick): SχPT
  - Mixed actions: MAχPT
- χPT expressions allow simultaneous extrapolations in m, L, and a,
- Predictions for discretization errors in spectrum of lattice Dirac operator
- Fitting forms for ϵ-regime

CAVEAT: need to truncate χPT$\Rightarrow$ additional systematic error

CAVEAT: $SU(2)$ vs $SU(3)$ χPT?

Aims of these lectures

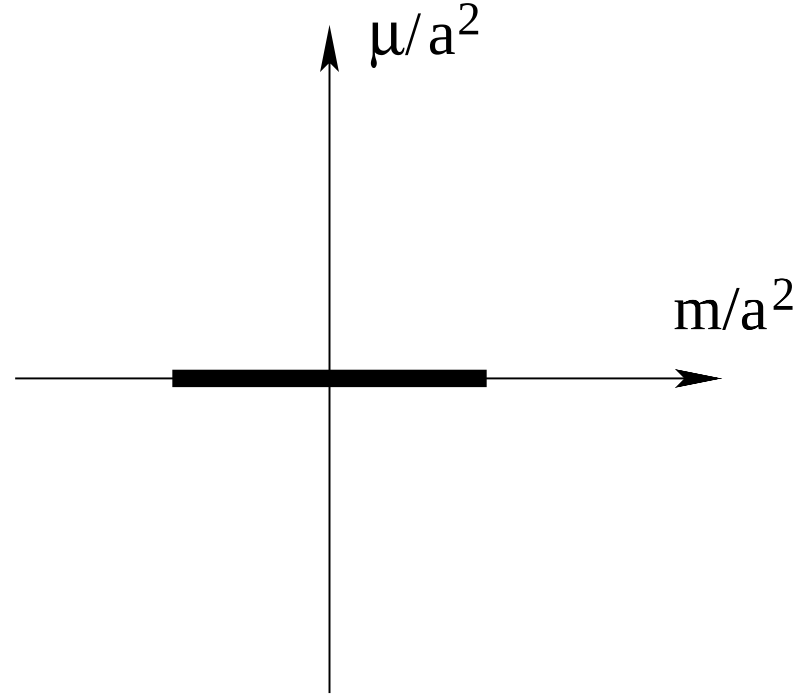
- Emphasize lessons for LQCD from χPT
- Provide introduction to "latticey" χPT
  - Use Wilson-tm fermions as example
- Provide introduction to PQχPT
- Show some applications
- Make you care about whether $m_u \longrightarrow 0$ is unambiguous!
  - If it is, then PQQCD is likely ill-defined

Outline of Lecture 1

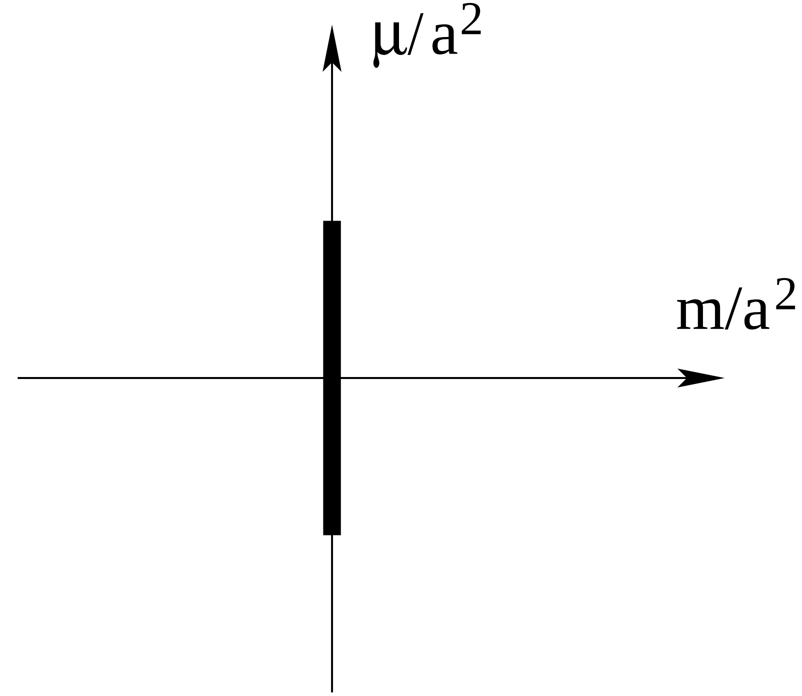
- Some highlights
- Review of continuum χPT
  - Focus on lessons for LQCD
  - Emphasize points relevant for subsequent generalizations
- Examples of results

Highlights I

Phase diagram for tmLQCD: tmχPT predicts two possibilities



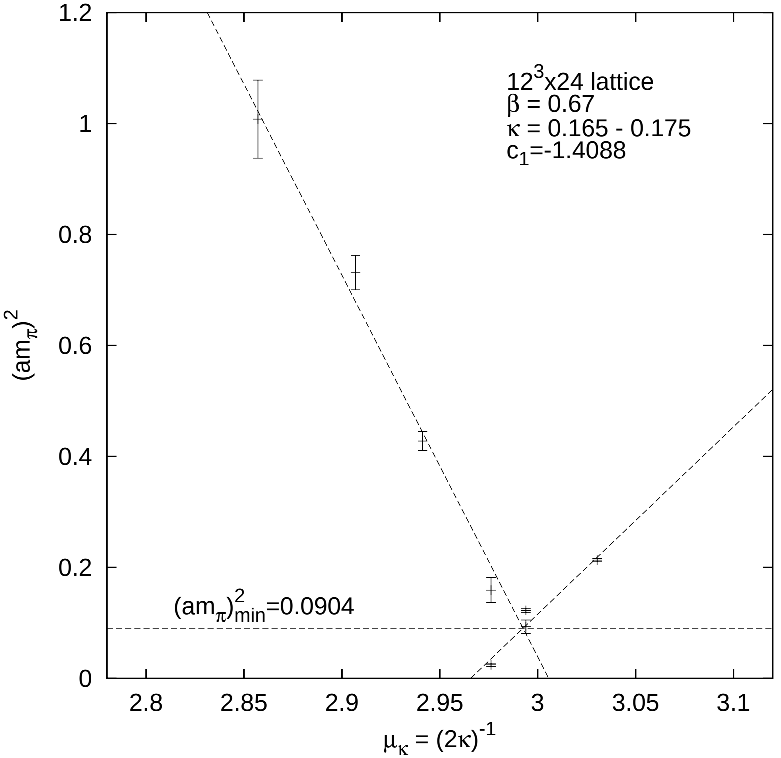
Aoki-phase along Wilson axis:
apparently holds for quenched theory, and dynamical fermions at large a
(Wilson gauge action)

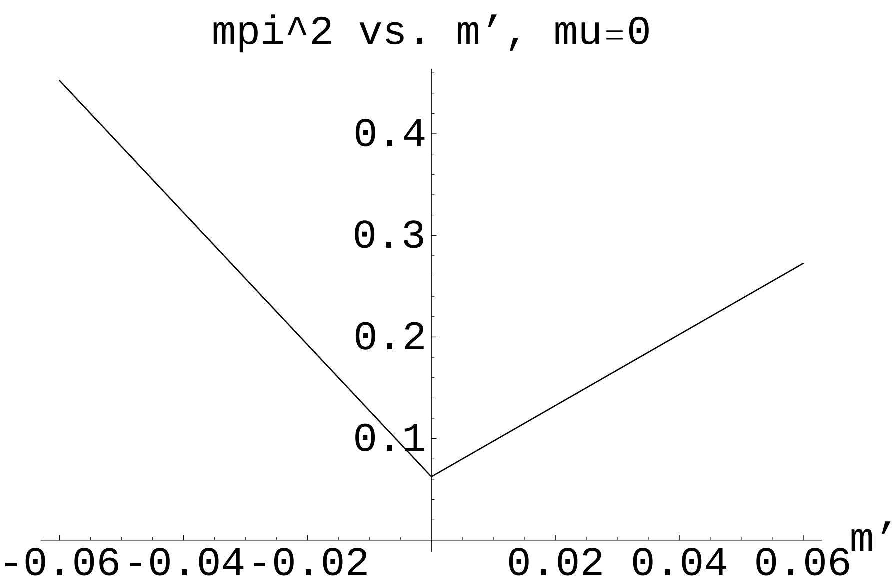


First-order phase transition:
apparently holds for dynamical fermions at small a
(Wilson or Symanzik gauge action)

Highlights II

Comparing [Farchioni *et al*,hep-lat/0410031] with tmχPT

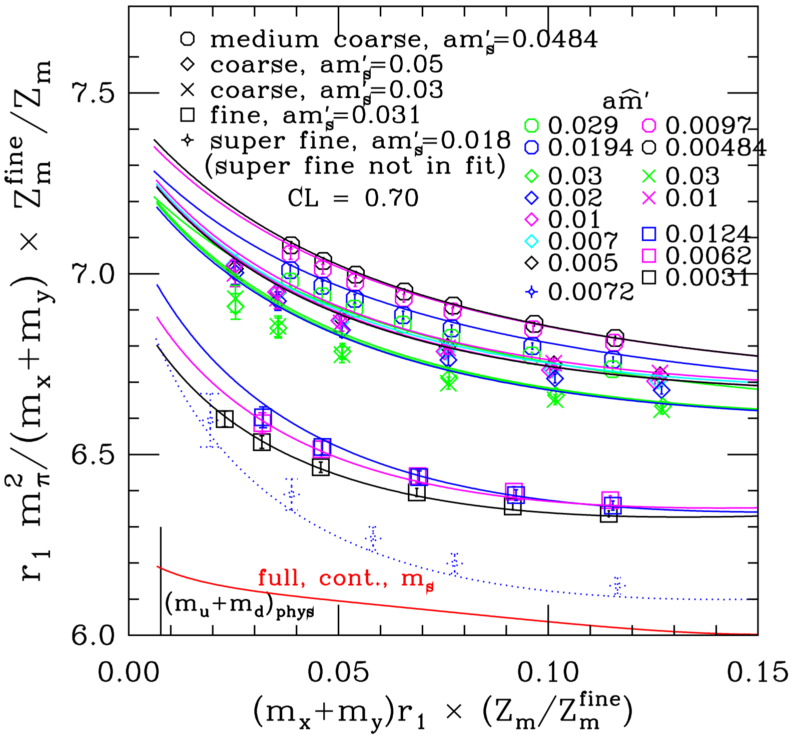


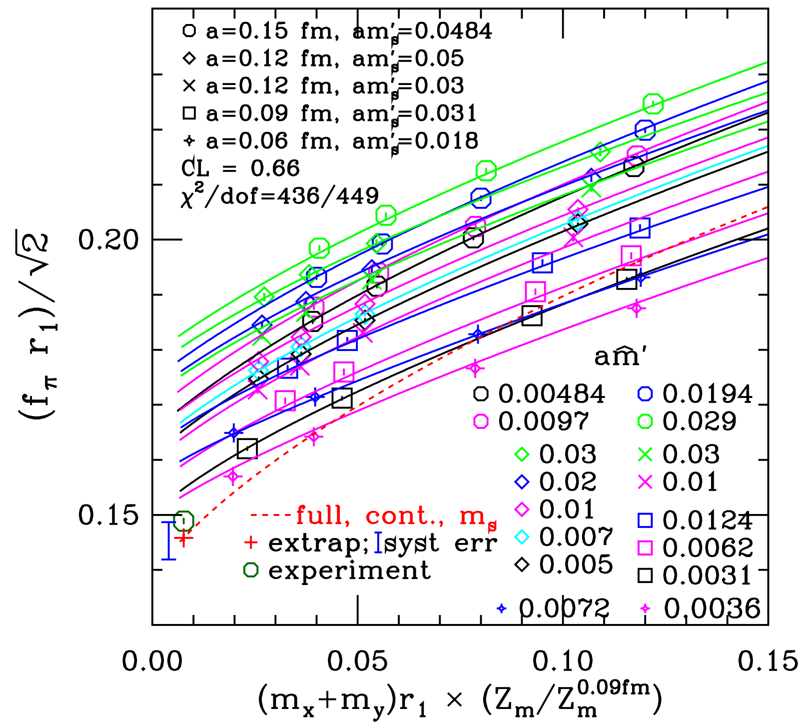


- Qualitative comparison only
- Difference in slopes for positive and negative m from 30% $O(a)$ contribution

Overview: Highlights III

Fitting staggered pion properties with SU(3) SχPT [MILC collaboration 07]

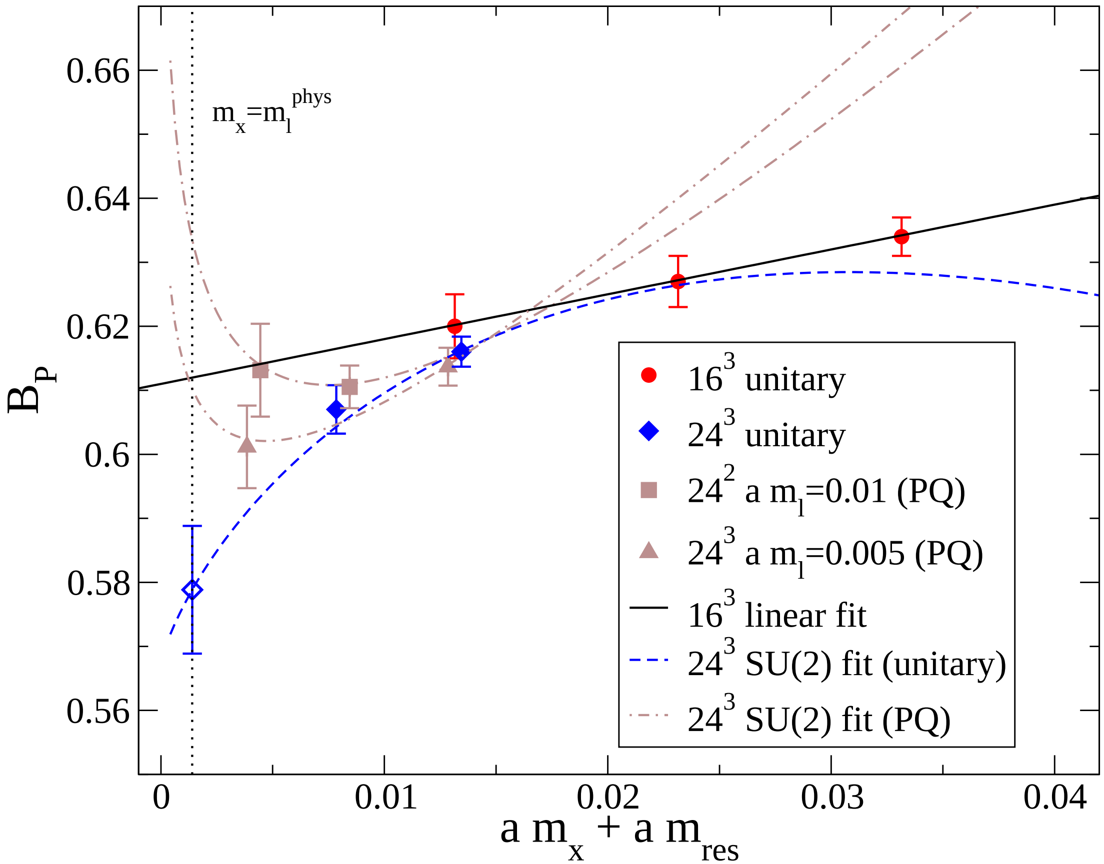




- $O(a^2)$ taste-breaking essential for fit
- PQ data essential to constrain parameters (e.g. 416 points/48 params)

Overview: Highlights IV

Fitting B_K with $SU(2)$ SχPT [RBC-UKQCD collaboration 08]



PQ data allows test of predicted chiral logarithm

Outline of Lecture 1

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- Examples of results

Chiral symmetry of QCD action

- Fermionic part of **Euclidean** Lagrangian in matrix notation:

$$\mathcal{L}_{QCD} = \overline{Q}_L \not{D} Q_L + \overline{Q}_R \not{D} Q_R + \overline{Q}_L M Q_R + \overline{Q}_R M^\dagger Q_L$$

  - $Q^{tr} = (u, d, s)$, $\overline{Q}_{L,R} = \overline{Q}_{L,R}(1 \pm \gamma_5)/2$, $Q_{L,R} = [(1 \mp \gamma_5)/2] Q_{L,R}$
- In the massless limit, have $\mathcal{G} = SU(3)_L \times SU(3)_R$ symmetry:
  - $Q_{L,R} \to U_{L,R} Q_{L,R}$ and $\overline{Q}_{L,R} \to \overline{Q}_{L,R} U^\dagger_{L,R}$, with $U_{L,R} \in SU(3)_{L,R}$
- Add in mass term, e.g. $M = \mathrm{diag}(m_u, m_d, m_s)$, $m_q \neq 0$
  - axial transformations $U_L = U_R^\dagger$ broken
  - vector $SU(3)$ subgroup ($U_L = U_R$) also broken, except if masses degenerate
- If treat M as complex "spurion" field then maintain full chiral symmetry
  - $M \to U_L M U_R^\dagger$, $M^\dagger \to U_R M^\dagger U_L$

Approximate chiral symmetry

- Chiral symmetry is useful if M is **small**:
  - What is small? $m_q \ll \Lambda_{QCD} \sim 300$ MeV
  - More precise criterion in χPT: $m_{\pi,K,\eta} \ll \Lambda_\chi \equiv 4\pi f_\pi \approx 1200$ MeV
  - $(m_u + m_d)/2 \approx 4$ MeV $\Rightarrow SU(2)_L \times SU(2)_R$ is a good approximate symmetry
  - $m_s \approx 100$ MeV or $m_{K,\eta} \approx \Lambda_\chi/2 \Rightarrow SU(3)_L \times SU(3)_R$ is much less good
- Important question for lattice applications of chiral perturbation theory and thus PQQCD:
  - Is m_s small enough that approximate chiral symmetry is useful to determine the quark mass dependence when $m_s^{\rm lat} \approx m_s$?
  - If not, then can only use chiral symmetry to guide extrapolations in m_u and m_d.

Spontaneous breaking of chiral symmetry

- Vacuum breaks chiral symmetry
  - No parity doubling in spectrum: $m_N(P=+) \neq m_N(P=-)$
  - Lightness of π, K and η consistent with their being pseudo-Goldstone bosons (PGBs)

- Order parameter

$$\langle \bar{q}q \rangle = \langle (\bar{q}_L q_R + \bar{q}_R q_L) \rangle \sim \Lambda^3_{\text{QCD}} \neq 0 \,, \quad q = u, d, s$$

  - Lattice simulations $\Rightarrow \langle \bar{q}q \rangle \neq 0$
  - Success of chiral perturbation theory

- Vector symmetry not spontaneously broken
  - If $m_u = m_d = m_s$ then $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle$
  - Based on experiment, and [Vafa-Witten] theorem

Symmetry breaking ($M = 0$)

- Condensate is LR flavor matrix:

$$\Omega_{ij} = \langle Q_{L,i,\alpha,c} \overline{Q}_{R,j,\alpha,c} \rangle \underset{\mathcal{G}}{\longrightarrow} U_L \, \Omega \, U_R^\dagger$$

  - All choices of Ω_{ij} are equivalent: "vacuum manifold"
  - Unbroken vector symmetry $\Rightarrow$ $\Omega_{ij} = \omega \, \delta_{ij}$ is in manifold
  - $\omega \neq 0$ implies chiral symmetry breaking:

$$\underbrace{SU(3)_L \times SU(3)_R}_{\mathcal{G}} \longrightarrow \underbrace{SU(3)}_{\mathcal{H}}$$

- Goldstone's theorem: 8 broken generators $\Rightarrow$ 8 GBs (π, K, η)

Symmetry breaking ($M \neq 0$)

- "Direction" of condensate depends on M:

$$\mathcal{V} = -\mathrm{tr}(\Omega M^\dagger) - \mathrm{tr}(\Omega^\dagger M)$$

- Conventional choice (M diagonal and positive) gives, when $M \to 0$,

$$\Omega_{ij} = \omega\, \delta_{ij}\,, \qquad \omega = -\langle \bar{q} q \rangle > 0$$

  - $\Omega = \omega \underset{\mathcal{G}}{\longrightarrow} \omega U_L U_R^\dagger \quad \Rightarrow \quad \mathcal{H} = SU(3)_V : U_L = U_R$
  - Axial tranformations $U_L = U_R^\dagger$ are broken
- A "twisted" mass, $M \to U_L M U_L$, rotates vacuum, $\Omega = \omega U_L^2$
  - All twists are physically equivalent

Building the effective field theory

- We have the correct ingredients for an EFT:
  - Separated scales $m_{\rm PGB} \ll \Lambda_\chi \sim m_\rho, m_{\rm nucleon}$
  - Maintain scale separation by considering $p_{\rm GB} \ll 1{\rm GeV}$
- Build EFT using **only GB fields, static sources, and spurions (M)**
  - Construct most general local Lagrangian consistent with symmetries
  - Non-renormalizable, many unknown LECs (low energy constants)
  - Gives most general unitary S-matrix consistent with symmetries [Weinberg]
- Order terms using power-counting
  - Small parameter is $p^2/\Lambda_\chi^2 \sim M/\Lambda_{\rm QCD}$

Representing GB fields

- Conceptually most non-trivial step of construction:
  - EFT built from GBs (mesons), while QCD built from quarks
  - Choice of GB fields not unique (not discussed here)
- "Promote" condensate to a field of fixed modulus:

$$\frac{\Omega_{ij}}{\omega} \equiv \frac{\langle Q_{L,i,\alpha,c}\overline{Q}_{R,j,\alpha,c}\rangle}{|\langle \bar{q}q\rangle|} \equiv \Sigma_{ij} \longrightarrow \Sigma_{ij}(x) \in SU(3)$$

- Tranforms under $\mathcal{G} = SU(3)_L \times SU(3)_R$ like Ω (i.e. linearly):

$$\Sigma(x) \underset{\mathcal{G}}{\longrightarrow} U_L \Sigma(x) U_R^\dagger$$

- Any VEV of Σ breaks $\mathcal{G}$ to $\mathcal{H} = SU(3)$ $\Rightarrow$ desired symmetry breaking
- Fluctuations correspond to GB (pion) fields. E.g. if $\langle\Sigma\rangle = 1$

$$\Sigma(x) = \exp\left(2i\pi^a(x)T^a/f\right) , \qquad a = 1, 8$$

- GB fields transform non-linearly under $\mathcal{G}$

Building $\mathcal{L}_{\rm eff}$

[Gasser & Leutwyler]

- Ingredients are Σ, $\Sigma^\dagger$, M, $M^\dagger$ and external sources (ℓ_μ, r_μ, s, p)

$$\begin{aligned} \Sigma &\to U_L \Sigma U_R^\dagger\,, \qquad \Sigma^\dagger \to U_R \Sigma^\dagger U_L^\dagger\,, \\ D_\mu\Sigma &= \partial_\mu\Sigma - i l_\mu \Sigma + i\Sigma r_\mu \to U_L (D_\mu \Sigma) U_R^\dagger\,, \\ M &\to s + ip \\ M &\to U_L M U_R^\dagger\,, \qquad M^\dagger \to U_R M^\dagger U_R^\dagger\,, \end{aligned}$$

- Write all terms that are local and $SU(3)_L \times SU(3)_R$, Euclidean, C & P invariant (simplifying using $\Sigma\Sigma^\dagger = 1$ etc.)
- Order in powers of $\partial^2 \sim M$
- **Skipping details . . .**

Leading order Lagrangian

- At leading order have:

$$\mathcal{L}^{(2)} = \frac{f^2}{4}\mathrm{tr}\left(D_\mu\Sigma D_\mu\Sigma^\dagger\right) - \frac{f^2 B_0}{2}\mathrm{tr}(M\Sigma^\dagger + \Sigma M^\dagger)$$

- Two unknown LECs: f and B_0
  - Expect $f \sim B_0 \sim \Lambda_{\mathrm{QCD}}$
- Set M to physical value:

$$M = \mathrm{diag}(m_u, m_d, m_s) = M^\dagger$$

- Determine VEV $\langle\Sigma\rangle$ by minimizing potential:

$$\mathcal{V}^{(2)} = -\frac{f^2 B_0}{2}\mathrm{tr}\left(M[\Sigma^\dagger + \Sigma]\right)$$

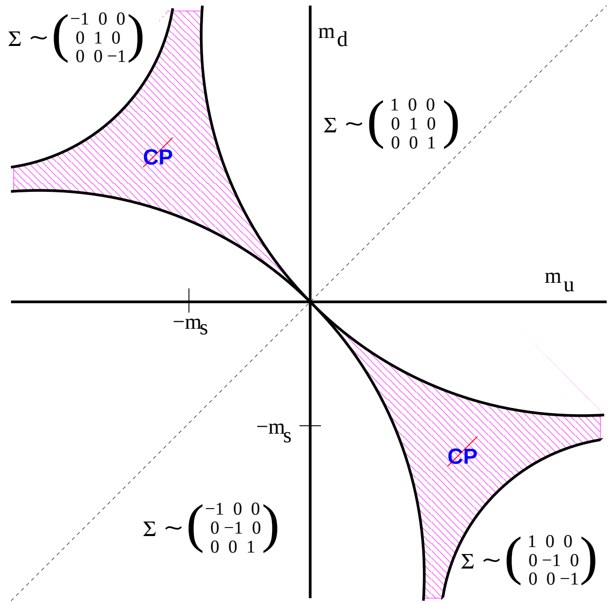
- If all $m_q > 0$, find $\langle\Sigma\rangle = 1$

Aside on vacuum structure

$$\mathcal{V}^{(2)} = -\frac{f^2 B_0}{2} \mathrm{tr}\left(M[\Sigma^\dagger + \Sigma]\right)$$

- For two flavors:
  - If we use $\langle\Sigma\rangle = \exp(i\theta\vec{n}\cdot\vec{\tau})$, then $\langle[\Sigma^\dagger + \Sigma]\rangle = 2\cos\theta \times 1$
  - Thus $\mathcal{V}^{(2)} \propto -\mathrm{tr}(M)\cos\theta$
  - So if $\mathrm{tr}M > 0$, $\langle\Sigma\rangle = 1$, while if $\mathrm{tr}M < 0$, $\langle\Sigma\rangle = -1$
  - ⇒ For degenerate quarks, have first order phase transition at $m = 0$

- For three flavors, $\Sigma = -1$ not possible
  - Interesting phase structure if some $m_q < 0$ [Dashen,Creutz]
  - $m_u = 0$ is not special if $m_d \neq 0$: no subgroup of $SU(3)_L \times SU(3)_R$ is restored



$m_s > 0$ fixed

Leading order (P)GB properties

- Insert $\Sigma = \exp(2i\pi/f)$, with $\pi \equiv \pi^a T^a$, into leading order (LO) $\mathcal{L}$

$$\mathcal{L}^{(2)} = \frac{f^2}{4}\mathrm{tr}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right) - \frac{f^2 B_0}{2}\mathrm{tr}(M[\Sigma^\dagger + \Sigma])$$
$$= \mathrm{tr}(\partial_\mu\pi\partial_\mu\pi) + 2B_0\,\mathrm{tr}(M\pi^2)$$
$$+\frac{1}{3f^2}\mathrm{tr}([\pi,\partial_\mu\pi][\pi,\partial_\mu\pi]) - \frac{2B_0}{3f^2}\mathrm{tr}(M\pi^4) + O(\pi^6)$$

  - $m^2_{\mathrm{PGB}} \propto M$
    - For degenerate quarks, $m_\pi^2 = 2B_0 m_q$
    - B_0 related to condensate: $B_0 = -\langle\bar{q}q\rangle/f^2$
  - Matching currents $\Rightarrow$ $f = f_\pi$
  - Sequence of non-renormalizable interactions involving even numbers of PGBs, size determined by f and $B_0 M$

    $\Rightarrow$ LO χPT predictive: e.g. 6 pion interactions given by 4 pion term

LO mass predictions for real QCD

- Determine physical particles using $U(3)_V$ ($\pi \longrightarrow U_V\ \pi\ U_V^\dagger$)

$$\pi = \begin{pmatrix} \frac{\pi^0}{2} + \frac{\eta}{\sqrt{12}} & \frac{\pi^+}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\ \frac{\pi^-}{\sqrt{2}} & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{12}} & \frac{K^0}{\sqrt{2}} \\ \frac{K^-}{\sqrt{2}} & \frac{\overline{K^0}}{\sqrt{2}} & -\frac{2\eta}{\sqrt{12}} \end{pmatrix}$$

- Inserting into $-2B_0\mathrm{tr}(M\pi^2)$ find
  - Charged particle masses are simple: $m^2_{q_i q_j} = B_0(m_i + m_j)$, $i \neq j$

$$\Rightarrow \frac{m^2_{K^+} + m^2_{K^0}}{2m^2_{\pi^+}} = \frac{m_\ell + m_s}{2m_\ell} + \mathrm{EM} \approx 13 \qquad \left(m_\ell = \frac{m_u + m_d}{2}\right)$$

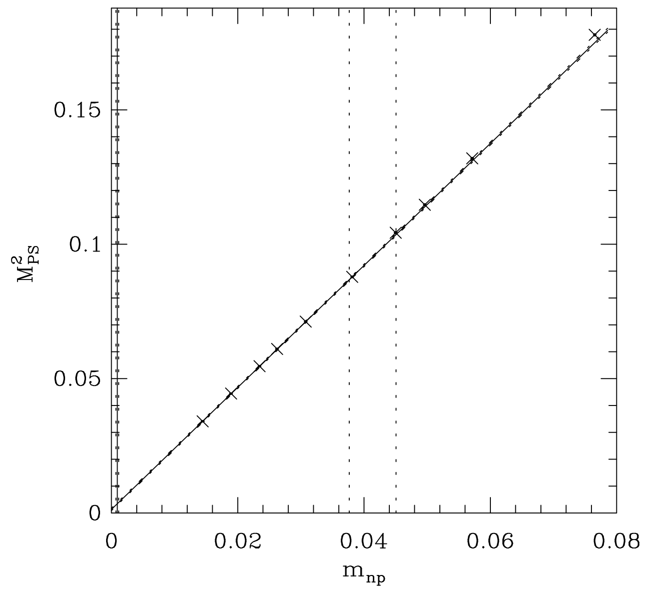
  - π^0 and η mix, but with small angle $\theta \sim (m_u - m_d)/m_s \ll 1$

$$\begin{aligned} m^2_{\pi^0} &= m^2_{\pi^+} + O(\theta^2 m_K^2) + \mathrm{EM}\,, \\ \underbrace{m^2_\eta}_{(548\ \mathrm{MeV})^2} &= \underbrace{(2[m^2_{K^+} + m^2_{K^0}] - m^2_{\pi^+})/3}_{(566\ \mathrm{MeV})^2} + O(\theta^2 m_K^2) \end{aligned}$$

- Cannot determine quark masses from χPT since scale dependent
  - Always appear in combination $\chi \equiv 2B_0 M$ which I use below

Lessons for lattice simulations (I)

+ LO χPT works to $\sim 10\%$ in GMO relation
  - Indeed, $m_{\pi^+}^2/m_q \sim \mathrm{const.}$ seen in all simulations (since 1983)
  - E.g. quenched Wilson fermions [Bhattacharya95]
    [vertical lines indicate $m_s^{\rm phys}$]

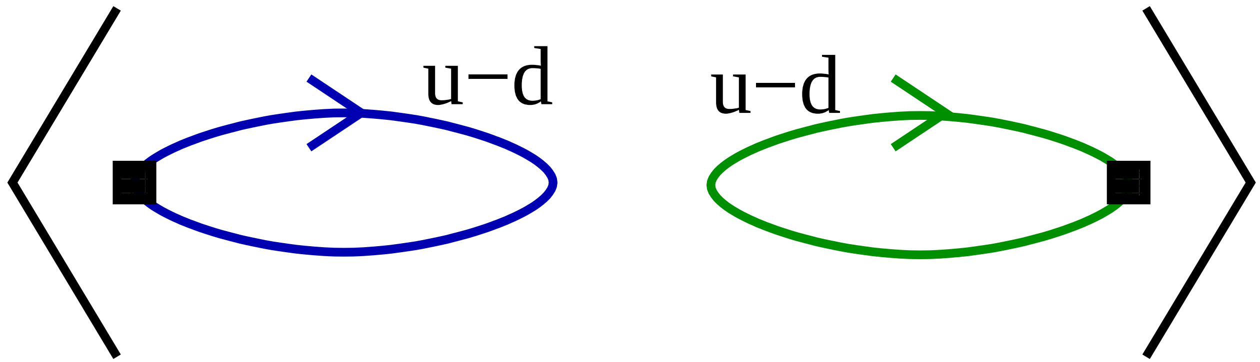


+ Can vary m_q in simulations (more "knobs to turn" than in real QCD), and χPT describes dependence on quark masses **in terms of the physical LECs**

Lessons for lattice simulations (II)

+ If simulate isospin limit $m_u = m_d$ then close to real QCD:
  - ▷ $m_u/m_d \sim 1/2$ does not lead to large isospin violations
  - ▷ Differences are suppressed by $(m_u - m_d)/m_s$ (PGBs) or by $(m_u - m_d)/\Lambda_{\rm QCD}$ (other hadrons)

− Calculating isospin breaking effects (e.g. $m^2_{\pi^+} - m^2_{\pi^0}$) is hard
  - ▷ Quark mass contributions involve disconnected diagrams and are small



  - ▷ EM contributions are comparable and not easy to calculate (but recent progress)

Next order chiral Lagrangian

- At NLO have 10 LECs and 2 “high-energy coefficients”:

$$\begin{aligned}\mathcal{L}^{(4)} \quad = \quad & -L_1\,\mathrm{tr}(D_\mu\Sigma D_\mu\Sigma^\dagger)^2 - L_2\,\mathrm{tr}(D_\mu\Sigma D_\nu\Sigma^\dagger)\mathrm{tr}(D_\mu\Sigma D_\nu\Sigma^\dagger) \\ & +L_3\,\mathrm{tr}(D_\mu\Sigma D_\mu\Sigma^\dagger D_\nu\Sigma D_\nu\Sigma^\dagger) \\ & +L_4\,\mathrm{tr}(D_\mu\Sigma^\dagger D_\mu\Sigma)\mathrm{tr}(\chi^\dagger\Sigma+\Sigma^\dagger\chi) + L_5\,\mathrm{tr}(D_\mu\Sigma^\dagger D_\mu\Sigma)[\chi^\dagger\Sigma+\Sigma^\dagger\chi]) \\ & -L_6\left[\mathrm{tr}(\chi^\dagger\Sigma+\Sigma^\dagger\chi)\right]^2 - L_7\left[\mathrm{tr}(\chi^\dagger\Sigma-\Sigma^\dagger\chi)\right]^2 - L_8\,\mathrm{tr}(\chi^\dagger\Sigma\chi^\dagger\Sigma+\mathrm{p.c.}) \\ & +L_9\,i\mathrm{tr}(L_{\mu\nu}D_\mu\Sigma D_\nu\Sigma^\dagger + p.c.) + L_{10}\,\mathrm{tr}(L_{\mu\nu}\Sigma R_{\mu\nu}\Sigma^\dagger) \\ & +H_1\,\mathrm{tr}(L_{\mu\nu}L_{\mu\nu}+p.c.) + H_2\,\mathrm{tr}(\chi^\dagger\chi)\end{aligned}$$

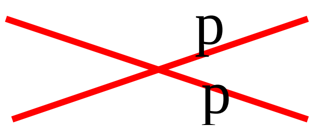
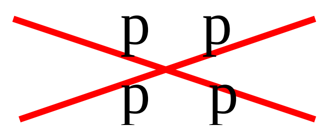
- L_i are “Gasser-Leutwyler coefficients”
  - Fundamental parameters of QCD, akin to hadron mass ratios
  - A subset can be determined experimentally to good accuracy
  - A different subset is straightforward to determine on the lattice
- $H_{1,2}$ give contact terms in correlation functions
- At NNLO there are 90 LECs and 4 HECs! [Bijnens et al]

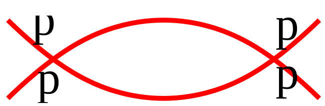
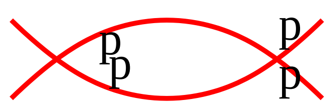
Power counting in χPT ($M = 0$)

$$\mathcal{L}^{(2)} \sim (\partial\pi)^2 + \frac{\pi^2(\partial\pi)^2}{f^2} + \ldots$$

$$\mathcal{L}^{(4)} \sim L_{GL}\left[\frac{(\partial\pi)^4}{f^4} + \frac{\pi^2(\partial\pi)^4}{f^6}\right]$$

- Consider $\pi\pi$ scattering (with, say, dim. reg. to avoid power divergences):

$\mathcal{L}^{(2)}_{\rm tree}$:  $\sim \frac{p^2}{f^2}$ $\mathcal{L}^{(4)}_{\rm tree}$:  $\sim L_{GL}\left(\frac{p^2}{f^2}\right)^2$

$\mathcal{L}^{(2)}_{1-\rm loop}$:  $\sim$  $\sim \left(\frac{p^2}{f^2}\right)^2 \frac{\ln(p^2/\mu^2)}{(4\pi)^2}$

- Have expansion in p^2/f^2 (and $m^2_{\rm PGB}/f^2$) up to logs
  - Theory is **predictive** up to truncation errors
  - E.g. at LO, $\mathcal{A}(\pi\pi \to \pi\pi)$ predicted in terms of f, up to errors of relative size p^2/f^2
  - Only a finite number of diagrams and LECs at each order, so can always make predictions
  - Non-analytic behavior ("chiral logs") does not involve new LECs

True expansion parameter?

- LEC's run with μ:
  - $dL_{GL}/d\ln(\mu) \approx 1/(4\pi)^2 \Rightarrow L_{GL}(2\mu) - L_{GL}(\mu) \approx 1/(4\pi)^2$
- So guess ("naive dimensional analysis"):
  - $L_{GL}(\mu \approx m_\rho) \approx 1/(4\pi)^2$
- Works well phenomenologically: $-1 \lesssim L_{GL}(4\pi)^2 \lesssim +1$
- Implies expansion parameter is p^2/Λ_χ^2, with $\Lambda_\chi = 4\pi f$
- For $M \neq 0$, $p^2/\Lambda_\chi^2 \longrightarrow (p^2 \text{ or } m^2_{\rm PGB})/\Lambda_\chi^2$

Lessons for lattice simulations (III)

+ Use χPT to extend reach of lattice to multiparticle processes
  - Calculate LECs from lattice simulations using simple physical quantities (e.g. masses)
  - Use χPT + LECs to determine multiparticle processes (scattering amplitudes, $\pi\pi \to 4\pi$, etc.) that are difficult or impossible to determine directly using simulations
    - Determining $\mathcal{A}(K \to \pi\pi)$ using unphysical, but more accessible, matrix elements [Rome-Southampton, Laiho-Soni]

– Always have truncation error when using χPT
  - Need to include NNLO terms (at least approximately) to determine NLO coefficients (L_{GL})
  - Fitting requires (approximate) NNNLO coefficients to work up to $m_s^{\rm phys}$ [MILC]

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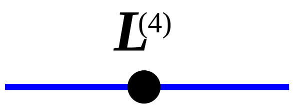
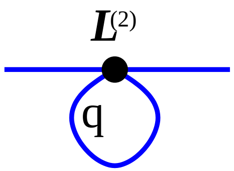
Results from χPT at NLO

- Charged PGB masses:
  - LO: $m^2_{\mathrm{PGB},0} = (\chi_{q1} + \chi_{q2})/2 = 2B_0(m_{q1} + m_{q2})/2$
  - NLO–tree:

$\delta m^2_{\mathrm{PGB}} \sim$ 

$\sim \chi\, L\, \frac{\chi}{f^2} \sim \chi(16\pi^2 L)\frac{m^2_{\mathrm{PGB},0}}{\Lambda^2_\chi}$

  - NLO–loop:

$\delta m^2_{\mathrm{PGB}} \sim$ 

$\sim \frac{\chi}{f^2}\int_q \frac{1}{q^2+m^2_{\mathrm{PGB}}} \sim \chi\frac{m^2_{\mathrm{PGB},0}}{\Lambda^2_\chi}\ln\left(\frac{m^2_{\mathrm{PGB},0}}{\mu^2}\right)$

$$m^2_{\pi^\pm} = \chi_\ell\left\{1 + \frac{8}{f^2}[\underbrace{(2L_8 - L_5)\chi_\ell}_{\text{valence}} + \underbrace{(2L_6 - L_4)(2\chi_\ell + \chi_s)]}_{\text{sea}} + \underbrace{\frac{3L_\pi - L_\eta}{6}}_{\text{logs}}\right\}$$

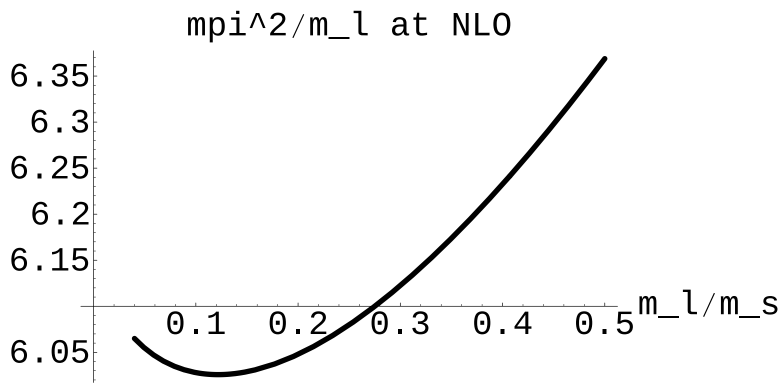
$$L_\pi = \frac{m^2_\pi}{\Lambda^2_\chi}\ln\left(\frac{m^2_\pi}{\mu^2}\right), \qquad L_\eta = \frac{m^2_\eta}{\Lambda^2_\chi}\ln\left(\frac{m^2_\eta}{\mu^2}\right)$$

Lessons for lattice simulations (IV)

$$m^2_{\pi^\pm} = \chi_\ell \left\{ 1 + \frac{8}{f^2} [\underbrace{(2L_8 - L_5)\chi_\ell}_{\text{valence}} + \underbrace{(2L_6 - L_4)(2\chi_\ell + \chi_s)}_{\text{sea}}] + \underbrace{\frac{3L_\pi - L_\eta}{6}}_{\text{logs}} \right\}$$

- Non-analytic terms important at small masses

  $m_s = 0.08$ GeV, $f = 0.093$ GeV, $L_5 = 1.45 \times 10^{-3}$, $L_8 = 10^{-3}$, $L_4 = L_6 = 0$
  [Bijnens, hep-ph/0409068]

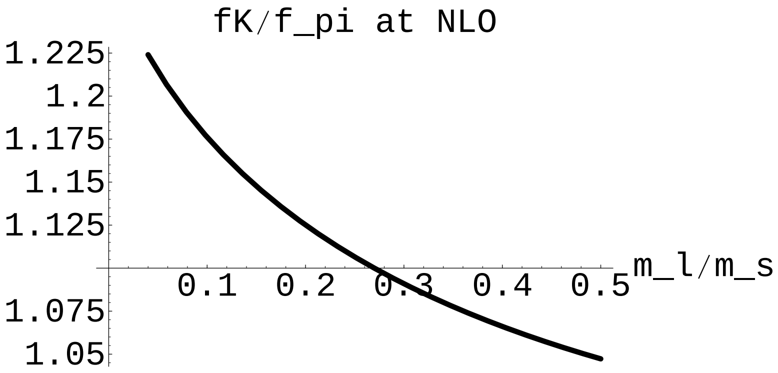


- **Must see chiral logs to have convincing results**
- Using PQ simulations allows separation of L_i

Further examples of chiral logs

$$\frac{f_K}{f_\pi} = 1 + \frac{2}{f^2}\underbrace{(L_5)(\chi_s - \chi_\ell)}_{\text{valence}} + \underbrace{\frac{5}{8}L_\pi - \frac{1}{4}L_K - \frac{3}{8}L_\eta}_{\text{logs}}$$

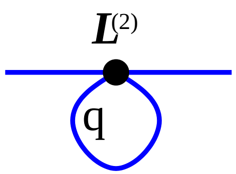
- Non-analytic terms important at small masses
  $m_s = 0.08$ GeV, $f = 0.085$ GeV,
  $L_5 = 1.45 \times 10^{-3}$, $L_4 = 0$



- Good to use "Golden Ratios" in which chiral logs cancel [Becirevic03,04]
- Some quantities have enhanced chiral logs, e.g. $\langle r^2\rangle_\pi \sim \ln(m_\pi^2/\mu^2)$

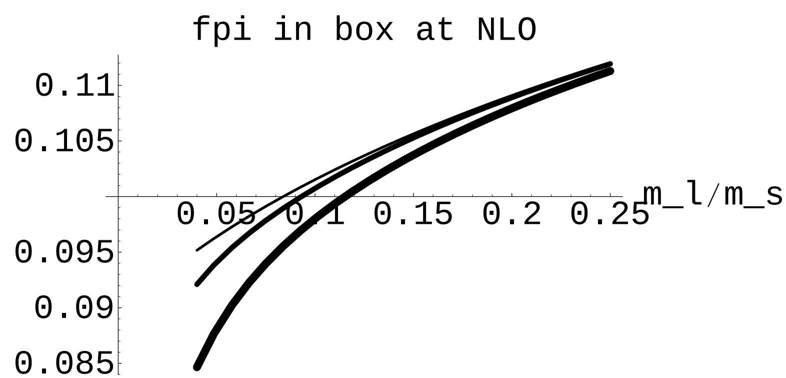
Volume dependence from χPT

- For **single particle matrix elements** pion (or more generally, PGB) loops give leading finite volume correction [Gasser+Leutwyler]
- Predicted along with coefficient of chiral log:
  - Replace momentum integral with sum



$$\rightarrow \int_q \left(\frac{1}{q^2+m_{\rm PGB}^2}\right) \rightarrow \int_{q_4} \sum_{\vec{q}=2\pi\vec{n}/L} \left(\frac{1}{q^2+m_{\rm PGB}^2}\right)$$

  - E.g. f_π at $a = 0.1$ fm, with $L = 2.4$ fm (thick), 3.2 fm and ∞
    $m_s = 0.08$ GeV, $f = 0.08$ GeV, $L_5 = 1.45 \times 10^{-3}$, $L_4 = 0$



- Formulae extended to higher order for some quantities [Lüscher, Colangelo]
  - LO contribution only trustworthy as indicator of size of FV effect
- Inclusion of volume dependence in χPT fits is now standard

Other quantities involving PGBs

- $SU(2)$ χPT complete at NNLO, including electroweak interactions
- Several predictions despite 53 LECs at NNLO (excluding electroweak)!
- Many quantities relevant for lattice simulations, e.g.
  - Pion scattering amplitude
  - Form factors of PGBs (vector and scalar)
  - Semileptonic form factors ($K \to \pi$)
  - B_K, $K \to \pi\pi$
- $SU(3)$ χPT (including electroweak) largely extended to NNLO
- Convergence?. [Bijnens, hep-ph/0401039,hep-ph/0409068]
  - $a_0^0(\pi\pi \to \pi\pi) = \underbrace{0.159}_{\text{LO}} + \underbrace{0.044}_{\text{NLO}} + \underbrace{0.016}_{\text{NNLO}} = 0.219 \pm ?$ c.f. $0.220(5)$
  - $f_K/f_\pi = \underbrace{1}_{\text{LO}} + \underbrace{0.169}_{\text{NLO}} + \underbrace{0.051}_{\text{NNLO}}$ (fit)
  - But for m^2_{PGB}, NNLO terms larger than NLO

Extension to “heavy” particles

- Heavy-light mesons in $1/m_B$ expansion [Wise, Burdman & Donoghue]

$$F_B \sim F_{B,0}(1 + \underbrace{m_\pi^2}_{\text{analytic}} + \underbrace{m_\pi^2 \ln(m_\pi)}_{\text{chiral log}} + \dots)$$

  - ▷ Similar expansion to those for PGB properties
  - ▷ Non-analytic terms involve additional coefficient $g_{\pi BB^*}$
  - ▷ Presence of nearby B^* invalidates $SU(3)$ χPT[Becirevic *et al*]

- Baryons [Jenkins & Manohar] and Vector mesons [Jenkins et al]

$$M_H \sim M_0 + \underbrace{m_\pi^2}_{\text{analytic}} + \underbrace{g_{\pi HH'}\, m_\pi^3}_{\text{leading loop}} + \underbrace{m_\pi^4 \ln(m_\pi)}_{\text{subleading loop}} + m_\pi^4 + \dots$$

  - ▷ Non-analytic terms involve additional coefficients (e.g. $g_{\pi NN}$)
  - ▷ Expansion in powers of m_π/Λ_χ (c.f. $(m_\pi/\Lambda_\chi)^2$ for mesons)
  - ⇒ **Poorer convergence**
  - ▷ (Improve using “finite range regularization”? [Leinweber et al])

Effective Field Theory/χPT: references

- A selection of books and lecture notes:
  - H. Georgi, "Weak Interactions and Modern Particle Theory"
  - J.F. Donoghue, E. Golowich and B.R. Holstein, "Dynamics of the Standard Model"
  - A.V. Manohar, "Effective Field Theories", hep-ph/9606222
  - G. Ecker, "Chiral Perturbation Theory", hep-ph/9608226,9805300
  - A. Pich, "Introduction to Chiral Perturbation Theory", hep-ph/9502366
  - D.B. Kaplan, "5 lectures on Effective Field Theory", nucl-th/0510023
- Classic papers:
  - S.R. Coleman, J. Wess and B. Zumino, "Structure Of Phenomenological Lagrangians. 1," Phys. Rev. **177**, 2239 (1969).
  - C.G. Callan, S.R. Coleman, J. Wess and B. Zumino, "Structure Of Phenomenological Lagrangians. 2," Phys. Rev. **177**, 2247 (1969).
  - S. Weinberg, "Phenomenological Lagrangians," PhysicaA **96**, 327 (1979).
  - J. Gasser and H. Leutwyler, "Chiral Perturbation Theory To One Loop," Annals Phys. **158**, 142 (1984).
  - J. Gasser and H. Leutwyler, "Chiral Perturbation Theory: Expansions In The Mass Of The Strange Quark," Nucl. Phys. B **250**, 465 (1985).