Non-perturbative *HQET* at the 1/m order

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ALPHA
Collaboration

Introduction

- Quark masses are very different in the standard model (from a few MeV to 200 GeV)
- Precise determinations of F_B and F_{B_s} are needed to constraint the free parameters of the SM.
- m_{b} free parameter
- In a heavy-light system it is very challenging to simulate a b-quark directly from the QCD Lagrangian.
- One can use an effective theory $\longrightarrow$ HQET is a natural choice for a B meson
- Computation of the $1/m$ corrections to have a better handle on the approximation

Heavy-light meson on the lattice

Heavy-light meson contains both light and heavy degrees of freedom
$m_s \sim 100\ \mathrm{MeV}$ and $m_b \sim 4\ \mathrm{GeV}$
$\Rightarrow$ need a large volume and a small lattice spacing

- Bare heavy quark mass $am_\mathrm{b} \ll 1$, eg $am_\mathrm{b} = 0.1$
  $\Rightarrow$ For a O(a)-improved action, leading discr error $\mathcal{O}(am_\mathrm{b})^2 \sim 1\%$

- Spatial extent $L = aN$. For instance impose $L > 2\ \mathrm{fm}$
  $\Rightarrow$ Requires a large number of points

$$N > \frac{2\ \mathrm{fm}}{a} = (2\ \mathrm{fm}) \times (10 m_\mathrm{b}) = 80\ \mathrm{GeV\ fm} \sim 400$$

Not doable with nowadays computers $\Rightarrow$ Effective theory

Effective theories for heavy quark

Momentum of a heavy quark (inside a hadron) $p = m_Q v + k$
Interaction with light dof $k \sim \Lambda_{\mathrm{QCD}} \ll m_Q$
Separate the higher and lower components of the heavy quark, and find an effective lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \bar{\psi}_{\mathrm{h}}(x) \left[iv.D + \frac{(iD_{\perp})^2}{2m_Q} + \frac{g\sigma.G}{4m_Q} + ... \right] \psi_{\mathrm{h}}(x)$$

Different choices:

- Expansion in $\Lambda_{\mathrm{QCD}}/m_Q$: HQET
- Expansion in v and $1/am_Q$: NRQCD
- Fermilab Method

Alternative "relativistic heavy quarks" [Aoki et al '01, Christ et al, Lin et al '06]

HQET on the lattice

Action of the effective theory on a lattice [Eichten & Hill]

$$S_{\mathrm{HQET}} = a^4 \sum_x \{\overline{\psi}_{\mathrm{h}}(x)[D_0 + \delta m]\psi_{\mathrm{h}}(x) + \sum_{\nu=1}^{n} \mathcal{L}^{(\nu)}(x)\}$$

with

$$\mathcal{L}^{(\nu)}(x) = \sum_i \omega_i^{(\nu)} \mathcal{L}_i^{(\nu)}(x) \qquad\qquad \omega_i^{(\nu)} \propto (1/m)^\nu$$

At the $1/m$ order

$$\begin{aligned} \mathcal{L}_1^{(1)} &= \overline{\psi}_{\mathrm{h}}(-\sigma \cdot \mathbf{B})\psi_{\mathrm{h}} & \mathcal{L}_2^{(1)} &= \overline{\psi}_{\mathrm{h}}(-\tfrac{1}{2}\mathbf{D}^2)\psi_{\mathrm{h}} \\ &\equiv -\mathcal{O}_{\mathrm{spin}} & &\equiv -\mathcal{O}_{\mathrm{kin}} \\ \omega_1^{(1)} &= \omega_{\mathrm{spin}} & \omega_1^{(2)} &= \omega_{\mathrm{kin}} \end{aligned}$$

Green functions

Under the path integral: expand in $1/m \Rightarrow \mathcal{L}^{(\nu)}(x)$ only as insertions

$$\begin{aligned}\langle \mathcal{O} \rangle &= \mathcal{Z}^{-1} \int \mathrm{D}\phi \, \mathrm{e}^{-S_{\mathrm{light}} - a^4 \sum_x \overline{\psi}_{\mathrm{h}}(x)[D_0 + \delta m]\psi_{\mathrm{h}}(x)} \, \mathcal{O} \\ & \qquad \times \left\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots \right\} \\ &\equiv \langle [1 - a^4 \sum_x \mathcal{L}^{(1)}(x)] \, \mathcal{O} \, \rangle^{\mathrm{stat}}\end{aligned}$$

$$\begin{aligned}\langle \mathcal{O} \rangle &= \langle \mathcal{O} \rangle^{\mathrm{stat}} + \omega_{\mathrm{kin}} \sum_x \langle \mathcal{O}\mathcal{O}_{\mathrm{kin}}(x) \rangle^{\mathrm{stat}} + \omega_{\mathrm{spin}} \sum_x \langle \mathcal{O}\mathcal{O}_{\mathrm{spin}}(x) \rangle^{\mathrm{stat}} \\ &= \langle \mathcal{O} \rangle^{\mathrm{stat}} + \omega_{\mathrm{kin}} \, \langle \mathcal{O} \rangle^{\mathrm{kin}} + \omega_{\mathrm{spin}} \, \langle \mathcal{O} \rangle^{\mathrm{spin}}\end{aligned}$$

Coefficients $\omega_i^{(\nu)}$, $\alpha_i^{(\nu)}$ have to cancel power divergences

Non perturbative HQET on the lattice [Heitger & Sommer 03]

Matching in a finite volume

The coefficients $\omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}, \ldots \omega_{N_{\mathrm{HQET}}}$ of HQET need to be fixed non perturbatively.
This is achieved by the matching with QCD

$$\Phi_i^{\mathrm{QCD}}(L_1) = \Phi_i^{\mathrm{HQET}}(L_1) \qquad i = 1, \ldots, N_{\mathrm{HQET}}$$

This requires to be able to simulate the heavy quark with finite mass.
$\Rightarrow$ In a small volume ($L_1 \simeq 0.4\ \mathrm{fm}$), with $am_{\mathrm{b}} \ll 1$.

Evolution to a large volume

The observables are evolved in a large volume within the effective theory

Strategy

QCD $(am_b \ll 1)$ | HQET

$L_1 \sim 0.4~\mathrm{fm}$ | $\Phi^{\mathrm{QCD}}(L_1, am_{\mathrm{b}})$ ← Matching → $\Phi^{\mathrm{HQET}}(L_1)$

↕

$L_2 = 2L_1$ | $\left[\Phi^{\mathrm{HQET}}(L_2) - \Phi^{\mathrm{HQET}}(L_1)\right]$

↕

$L_{\mathrm{inf}} = 2L_2$ | $\Phi(L_{\mathrm{inf}}, am_{\mathrm{b}})$ ↔ $\left[\Phi^{\mathrm{HQET}}(L_{\mathrm{inf}}) - \Phi^{\mathrm{HQET}}(L_2)\right]$

Experiment

b-quark mass, the static approximation

- Choose $\Phi^{\rm QCD}(L,M) = L\Gamma^{\rm QCD}(L,M)$, a 'finite volume meson mass'.
- At the leading order of HQET (static approximation)

In infinite volume $\quad m_{\rm B}(M) = E^{\rm stat} + m_{\rm bare}$

In finite volume $\quad \Gamma^{\rm QCD}(L_1,M) = \Gamma^{\rm stat}(L_1) + m_{\rm bare}$

- Use the matching in L_1 and introduce a intermediate volume L_2

$$m_{\rm B}(M) = \underbrace{E^{\rm stat} - \Gamma^{\rm stat}(L_2)}_{a\to 0} + \underbrace{\Gamma^{\rm stat}(L_2) - \Gamma^{\rm stat}(L_1)}_{a\to 0} + \underbrace{\Gamma^{\rm QCD}(L_1,M)}_{a\to 0}$$

- Solve (in the continuum) $m_{\rm B}(M_b) = m_{\rm B}^{exp}$

b quark mass, the $1/m$ correction

At the LO, in infinite volume

$$m_{\mathrm{B}} = \underbrace{E^{\mathrm{stat}} + m_{\mathrm{bare}}}_{\mathrm{LO}}$$

$\Rightarrow$ Need 1 observable Φ.

b quark mass, the $1/m$ correction

At the NLO, in infinite volume

$$m_\mathrm{B} = \underbrace{E^\mathrm{stat} + m_\mathrm{bare}}_{\mathrm{LO}} + \underbrace{\omega_\mathrm{kin} E^\mathrm{kin} + \omega_\mathrm{spin} E^\mathrm{spin}}_{\mathrm{NLO}}$$

$\Rightarrow$ Need 3 observables Φ_1, Φ_2, Φ_3.

b quark mass, the $1/m$ correction

At the NLO, in infinite volume

$$m_{\rm B} = \underbrace{E^{\rm stat} + m_{\rm bare}}_{\rm LO} + \underbrace{\omega_{\rm kin} E^{\rm kin} + \omega_{\rm spin} E^{\rm spin}}_{\rm NLO}$$

$\Rightarrow$ Need 3 observables Φ_1, Φ_2, Φ_3.

Or, consider the spin-averaged B meson $\Rightarrow \omega_{\rm spin}$ cancels

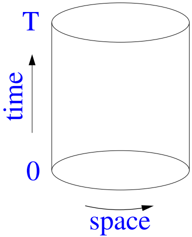
$$m_{\rm B}^{\rm av} \equiv \frac{1}{4} m_{\rm B} + \frac{3}{4} m_{\rm B}^{*} = E^{\rm stat} + m_{\rm bare} + \omega_{\rm kin} E^{\rm kin}$$

$\Rightarrow$ Need two observables Φ_1, Φ_2, and the spin splitting term becomes a separate issue.

Implementation : Schrödinger functional

Implementation: Schrödinger functional of size $T \times L^3$

- Dirichlet boundary conditions in time (at $x_0 = 0$ and $x_0 = T$)
- Periodic boundary conditions in space, up to a phase $\Psi(x + \hat{k}L) = e^{i\theta}\Psi(x)$.



Transition amplitude for $C(x_0 = 0) \rightarrow C'(x_o = T)$

$$\mathcal{Z}[C', C] = \langle C' | \mathrm{e}^{-\mathbb{H}\mathrm{T}} \mathbb{P} | \mathrm{C} \rangle$$

Correlators in the effective theory

Axial and vector (non-improved) current, in QCD

$$A_\mu(x) = \overline{\psi}_{\rm l}(x)\gamma_\mu\gamma_5\psi_{\rm b}(x)$$
$$V_\mu(x) = \overline{\psi}_{\rm l}(x)\gamma_\mu\psi_{\rm b}(x)$$

and in HQET

$$A_\mu^{\rm stat}(x) = \overline{\psi}_{\rm l}(x)\gamma_\mu\gamma_5\psi_{\rm h}(x)$$

With order-a improvement

$$(A_{\rm I})_\mu(x) = A_\mu(x) + c_{\rm A} A_\mu^{(1)}(x)$$
$$(V_{\rm I})_\mu(x) = V_\mu(x) + c_{\rm V} V_\mu^{(1)}(x)$$
$$(A_{\rm I}^{\rm stat})_\mu(x) = A_\mu^{\rm stat}(x) + c_{\rm A}^{\rm stat} A_\mu^{{\rm stat}\,(1)}(x)$$

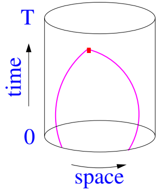
Note that $c_{\rm A}^{\rm stat} = \mathcal{O}(1/m)$

Implementation: 2 pts functions in QCD

Boundary to current correlators

$$f_{\rm A}(x_0) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle (A_{\rm I})_0(x)\,(\bar\zeta_{\rm b}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$

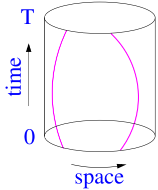
$$k_{\rm V}(x_0) = -\frac{a^6}{6}\sum_{\mathbf{y},\mathbf{z},k} \left\langle (V_{\rm I})_k(x)\,(\bar\zeta_{\rm b}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



and boundary to boundary correlator

$$f_1 = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar\zeta'_{\rm b}(\mathbf{y}')\gamma_5\zeta'_{\rm l}(\mathbf{z}'))\,(\bar\zeta_{\rm b}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$

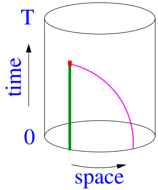
$$k_1 = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar\zeta'_{\rm b}(\mathbf{y}')\gamma_{\rm k}\zeta'_{\rm l}(\mathbf{z}'))\,(\bar\zeta_{\rm b}(\mathbf{y})\gamma_{\rm k}\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



Implementation: 2 pts functions in the static theory

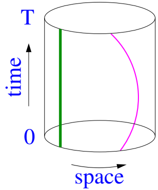
Boundary to current correlators

$$f_{\rm A}^{\rm stat}(x_0) \;=\; -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle (A_{\rm I}^{\rm stat})_0(x)\,(\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



and boundary to boundary correlator

$$f_1^{\rm stat} = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar\zeta'_{\rm h}(\mathbf{y}')\gamma_5\zeta'_{\rm l}(\mathbf{z}'))\,(\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



Heavy quark expansion

At the NLO of heavy quark effective theory

current to boundary correlators

$$f_{\rm A} \propto f_{\rm A}^{\rm stat} + c_{\rm A}^{\rm hqet} f_{\delta \rm A}^{\rm stat} + \omega_{\rm kin} f_{\rm A}^{\rm kin} + \omega_{\rm spin} f_{\rm A}^{\rm kin}$$

$$k_{\rm V} \propto f_{\rm A}^{\rm stat} + c_{\rm V}^{\rm hqet} f_{\delta \rm A}^{\rm stat} + \omega_{\rm kin} f_{\rm A}^{\rm kin} - \frac{1}{3}\omega_{\rm spin} f_{\rm A}^{\rm kin}$$

boundary to boundary correlators

$$f_1 \propto f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin} + \omega_{\rm spin} f_1^{\rm spin}$$

$$k_1 \propto f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin} - \frac{1}{3}\omega_{\rm spin} f_1^{\rm spin}$$

Remark on the strategy for the b-quark mass

Two different strategies:

- Use boundary to boundary correlators

  $\Rightarrow$ Need to fix $m_{\rm bare}$, $\omega_{\rm kin}$

  $\Rightarrow$ Need two observables.

- Use boundary to current correlators

  $\Rightarrow$ Need to fix $m_{\rm bare}$, $\omega_{\rm kin}$ and a L.C. of $c_{\rm A}^{\rm hqet}$ and $c_{\rm V}^{\rm hqet}$.

  $\Rightarrow$ Need three observables.

An example: elimination of $\omega_{\rm kin}$

- In QCD build the ratios $R_1^{\rm P} = \ln\left(\frac{f_1(\theta_1)}{f_1(\theta_2)}\right)$ and $R_1^{\rm V} = \ln\left(\frac{k_1(\theta_1)}{k_1(\theta_2)}\right)$
- Write down the corresponding expansions

$$\begin{aligned} R_1^{\rm P} &= R_1^{\rm stat} + \omega_{\rm kin} R_1^{\rm kin} + \omega_{\rm spin} R_1^{\rm spin} \\ R_1^{\rm V} &= R_1^{\rm stat} + \omega_{\rm kin} R_1^{\rm kin} - \frac{1}{3}\omega_{\rm spin} R_1^{\rm spin} \end{aligned}$$

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- Write down the corresponding expansions

$$
\begin{aligned}
R_1^{\rm P} &= R_1^{\rm stat} + \omega_{\rm kin} R_1^{\rm kin} + \omega_{\rm spin} R_1^{\rm spin} \\
R_1^{\rm V} &= R_1^{\rm stat} + \omega_{\rm kin} R_1^{\rm kin} - \frac{1}{3}\omega_{\rm spin} R_1^{\rm spin}
\end{aligned}
$$

- Then define the observables (in small volume L_1)

$$
\Phi_1 = \underbrace{\frac{1}{4}(R_1^{\rm P} + 3R_1^{\rm V})}_{R_1^{\rm av}} - \underbrace{R_1^{\rm stat}}_{\Phi_1^{\rm stat}} = \underbrace{\omega_{\rm kin} R_1^{\rm kin}}_{\mathcal{O}(1/m)}
$$

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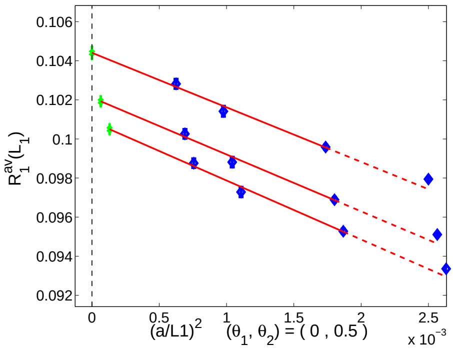
$$\Phi_1 = \underbrace{\frac{1}{4}(R_1^{\rm P} + 3R_1^{\rm V})}_{R_1^{\rm av}} - \underbrace{R_1^{\rm stat}}_{\Phi_1^{\rm stat}} = \underbrace{\omega_{\rm kin} R_1^{\rm kin}}_{\mathcal{O}(1/m)}$$

- Evolution : $\Phi_1(2L_1) = \underbrace{\frac{\Phi_1(2L_1)}{\Phi_1(L_1)}}_{HQET} \underbrace{\Phi_1(L_1)}_{\text{“}QCD\text{“}} = \underbrace{\left[\frac{R_1^{\rm kin}(2L_1)}{R_1^{\rm kin}(L_1)}\right]}_{\sigma_1} [\Phi_1(L_1)]$

How does it look like ?

QCD : $R_1^{av}(L_1)$



$L_1 M = 10.4, 12.1, 10.3$

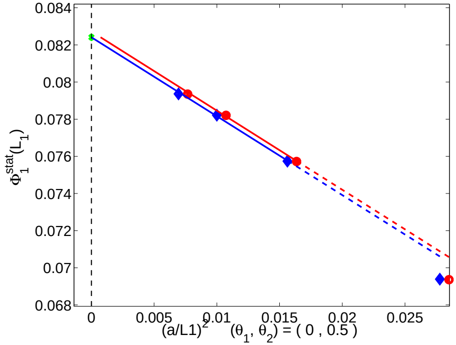
$L_1 \simeq 0.4$ fm

L_1/a	40	32	24	20
β	7.84	7.65	7.41	7.26

Cont extr in $(a/L)^2$

How does it look like ?

stat : Φ_1^{stat}



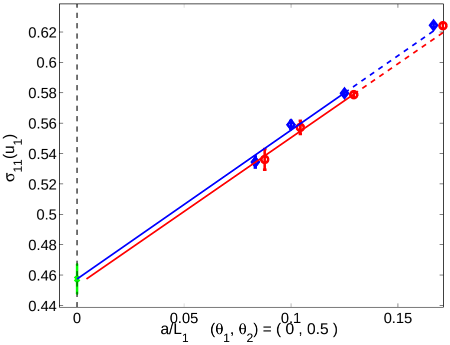
hyp1 hyp2

$L_1 \simeq 0.4$ fm

L_1/a	12	10	8	6
β	6.78	6.64	6.45	6.22

Cont extr in $(a/L)^2$

$1/m$: σ_{11}



hyp1 hyp2

$L_1 \simeq 0.4$ fm

L_1/a	12	10	8	6
β	6.78	6.64	6.45	6.22

$L_2 = 2L_1$

L_2/a	24	20	16	12
β	6.78	6.64	6.45	6.22

Cont extr in a/L

Observables for the b-quark mass

Define the observable Φ_1 such that

$$\Phi_1^{\mathrm{HQET}} = \omega_{\mathrm{kin}} \underbrace{\left\{ \frac{f_1^{\mathrm{kin}}(\theta)}{f_1^{\mathrm{stat}}(\theta)} - \frac{f_1^{\mathrm{kin}}(\theta')}{f_1^{\mathrm{stat}}(\theta')} \right\}}_{\mathrm{R}_1^{\mathrm{kin}}}$$

and Φ_2 such that

$$\Phi_2^{\mathrm{HQET}}(L) = L[m_{\mathrm{bare}} + \Gamma_1^{\mathrm{stat}} + \omega_{\mathrm{kin}} \Gamma_1^{\mathrm{kin}}]$$

b-quark mass at the $1/m$ order

- At the NLO of HQET $m_{\mathrm{B}} = E^{\mathrm{stat}} + m_{\mathrm{bare}} + \omega_{\mathrm{kin}} E^{\mathrm{kin}}$
- Matching 1 $\Phi_1^{\mathrm{QCD}}(L_1) = \omega_{\mathrm{kin}} R_1^{\mathrm{kin}}(L_1)$
- Matching 2 $\Phi_2^{\mathrm{QCD}}(L_1) = L_1 \left[\Gamma_1^{\mathrm{stat}}(L_1) + m_{\mathrm{bare}} + \omega_{\mathrm{kin}} \Gamma_1^{\mathrm{kin}}(L_1)\right]$
- Use the ssf

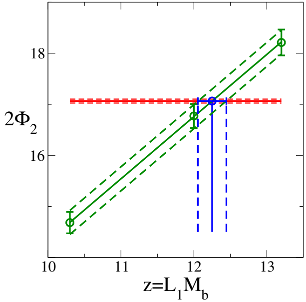
$$m_{\mathrm{B}} = \frac{\Phi_2^{\mathrm{QCD}}(L_1, M)}{L_1} + \left[E^{\mathrm{stat}} - \Gamma^{\mathrm{stat}}(L_2)\right] + \sigma^{\mathrm{stat}}$$

$$+ \left[\frac{\Phi_1^{\mathrm{QCD}}(L_1)}{R_1^{\mathrm{kin}}(L_2)} \sigma_1^{\mathrm{kin}} (E^{\mathrm{kin}} - \Gamma^{\mathrm{kin}}(L_1))\right]$$

- Solve (in the continuum) $m_{\mathrm{B}}(M_b) = m_{\mathrm{B}}^{exp}$.

$$L_2 m_{\rm B}(M) = 2\Phi_2^{\rm QCD}(L_1, M) + L_2\left[E^{\rm stat} - \Gamma^{\rm stat}(L_2)\right] + \sigma^{\rm stat}(u_1)$$

We solve $m_{\rm B}^{\rm stat}(M_{\rm b}^{\rm stat}) = m_{\rm B}^{\rm exp} = 5404\ {\rm MeV}$ by a linear interpolation



$$M_b^{\rm stat,RGI} = 6771 \pm 99\,{\rm MeV}$$

$m_{\rm b}$ including $1/m$ correciton

In the quenched approximation, find [Della Morte et al '05]

$$m_{\rm b}(m_{\rm b}) = \underbrace{4.350(64)}_{\rm static}\ {\rm GeV}\ \underbrace{-0.049(29)}_{{\rm O}(\Lambda^2/m_{\rm b})}\ {\rm GeV} + \underbrace{{\rm O}(\Lambda^3/m_{\rm b}^2)}_{\rm negligible}$$

Particle Data Group : $4.1 - 4.4\ {\rm GeV}$
Other lattice results :

4.41(5)(10)	[Martinelli & Sachrajda [98]]	NLO matching
4.30(5)(5)	[Martinelli & Sachrajda 98], [Lubicz 01]	NNLO matching

Observable(s) for the decay constant

Correlators: current-to-boundary f_A and boundary-to-boundary f_1

Build an observables related to the decay constant :

$$\Phi_B^{\mathrm{QCD}} = \ln \left(\frac{-f_{\mathrm{A}}(x_0)}{\sqrt{f_1}} \right) \quad \xrightarrow{L \gg 1} \quad \ln \left(\frac{1}{2} F_{\mathrm{B}} \sqrt{m_{\mathrm{B}} L^3} \right)$$

and in the effective theory (at the leading order):

$$\Phi_B^{\mathrm{hqet}} = \ln Z_{\mathrm{A}}^{\mathrm{stat}} + \ln \left(\frac{-f_{\mathrm{A}}^{\mathrm{stat}}}{\sqrt{f_1^{\mathrm{stat}}}} \right) + \mathcal{O}(1/m)$$

Strategy for F_{B_s} in the static approximation

The static heavy light decay constant

$$\Phi_B(L_\infty) = \left[\Phi_B^{\rm stat}(L_\infty) - \Phi_B^{\rm stat}(L_2)\right] + \left[\Phi_B^{\rm stat}(L_2) - \Phi_B^{\rm stat}(L_1)\right] + \Phi_B^{\rm QCD}(L_1)$$

Remarks

- $\ln Z_{\rm A}^{\rm stat}$ cancels out in the differences
- Terms in bracket have a continuum limit.
  But $L_\infty \simeq 1.5~\mathrm{fm}$ and $L_1 \simeq 0.4~\mathrm{fm}$
  $\longrightarrow$ Introduce a volume $L_2 = 2L_1$
- Step scaling functions $\sigma = \left[\Phi_B^{\rm stat}(L_2) - \Phi_B^{\rm stat}(L_1)\right]$
- Mass dependence from QCD

F_B, including $1/m$ corrections

At the LO of HQET

$$\Phi_B^{\mathrm{hqet}} = \ln Z_{\mathrm{A}}^{\mathrm{stat}} + \ln\left(\frac{-f_{\mathrm{A}}^{\mathrm{stat}}}{\sqrt{f_1^{\mathrm{stat}}}}\right) + c_{\mathrm{A}}^{\mathrm{stat}} \frac{f_{\delta\mathrm{A}}^{\mathrm{stat}}}{f_{\mathrm{A}}^{\mathrm{stat}}}$$

$\Rightarrow$ Need 2 observables

F_B, including $1/m$ corrections

At the NLO of HQET

$$\Phi_B^{\rm hqet} = \ln Z_{\rm A}^{\rm hqet} + \ln\left(\frac{-f_{\rm A}^{\rm stat}}{\sqrt{f_1^{\rm stat}}}\right) + c_{\rm A}^{\rm hqet}\frac{f_{\delta\rm A}^{\rm stat}}{f_{\rm A}^{\rm stat}} + \underbrace{\omega_{\rm kin}\left(\frac{f_{\rm A}^{\rm kin}}{f_{\rm A}^{\rm stat}} + \frac{1}{2}\frac{f_1^{\rm kin}}{f_1^{\rm stat}}\right) + \omega_{\rm spin}\left(\frac{f_{\rm A}^{\rm spin}}{f_{\rm A}^{\rm stat}} - \frac{1}{2}\frac{f_1^{\rm spin}}{f_1^{\rm stat}}\right)}_{1/m}$$

$\Rightarrow$ Need 4 observables

Observables for F_B at the $1/m$ order

Obs	QCD	stat	$1/m$
$\Phi_1 =$	$\frac{1}{4}(R_1^{\mathrm{P}} + 3R_1^{\mathrm{V}})$	$-R_1^{\mathrm{stat}}$	$= \omega_{\mathrm{kin}} R_1^{\mathrm{kin}}$
$\Phi_2 =$	$\frac{3}{4} \ln\left(\frac{f_1}{k_1}\right)$		$= \omega_{\mathrm{spin}} \frac{f_1^{\mathrm{spin}}}{f_1^{\mathrm{stat}}}$
$\Phi_3 =$	R_{A}	$-R_{\mathrm{A}}^{\mathrm{stat}}$	$= c_{\mathrm{A}}^{\mathrm{HQET}} R_{\delta\mathrm{A}} + \omega_{\mathrm{kin}} R_{\mathrm{A}}^{\mathrm{kin}} + \omega_{\mathrm{spin}} R_{\mathrm{A}}^{\mathrm{spin}}$

$$
\begin{aligned}
\Phi_B &= \ln Z_{\mathrm{A}}^{\mathrm{hqet}} + \ln\left(\frac{-f_{\mathrm{A}}^{\mathrm{stat}}}{\sqrt{f_1^{\mathrm{stat}}}}\right) + c_{\mathrm{A}}^{\mathrm{hqet}} \frac{f_{\delta\mathrm{A}}^{\mathrm{stat}}}{f_{\mathrm{A}}^{\mathrm{stat}}} \\
&+ \omega_{\mathrm{kin}} \left(\frac{f_{\mathrm{A}}^{\mathrm{kin}}}{f_{\mathrm{A}}^{\mathrm{stat}}} + \frac{1}{2}\frac{f_1^{\mathrm{kin}}}{f_1^{\mathrm{stat}}}\right) + \omega_{\mathrm{spin}} \left(\frac{f_{\mathrm{A}}^{\mathrm{spin}}}{f_{\mathrm{A}}^{\mathrm{stat}}} - \frac{1}{2}\frac{f_1^{\mathrm{spin}}}{f_1^{\mathrm{stat}}}\right)
\end{aligned}
$$

Strategy for F_B at the $1/m$ order (summary)

In a large volume, the decay constant is given by

$$\Phi_B(L_\infty) = \left[\Phi_B^{\rm hqet}(L_\infty) - \Phi_B^{\rm hqet}(L_2)\right] + \left[\Phi_B^{\rm hqet}(L_2) - \Phi_B^{\rm hqet}(L_1)\right] + \Phi_B^{\rm QCD}(L_1)$$

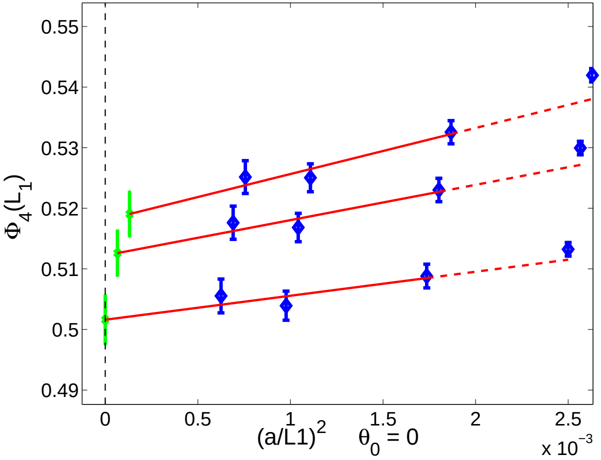
where the matching equations are used to eliminate the HQET parameters

$$\Phi_B^{\rm hqet}(2L_1) - \Phi_B^{\rm hqet}(L_1) = \sigma_{\rm stat}(u_1) + \underbrace{\sum_{i=1}^{3} \sigma_i(u_1)\Phi_i^{\rm QCD}(L_1)}_{\mathcal{O}(1/m)}$$

One can also separate the $1/m$ correction

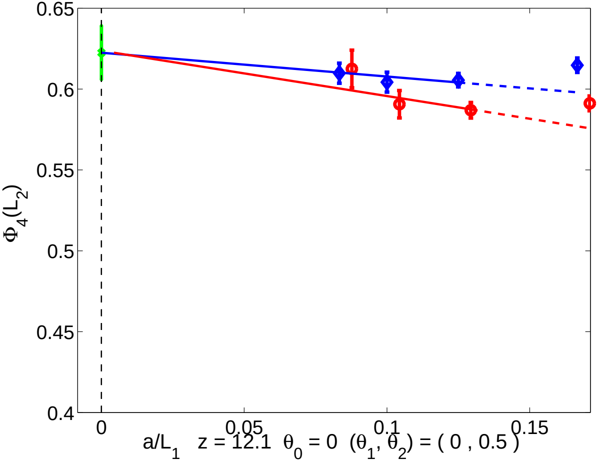
$$\Phi_B(L) = \Phi_B^{\rm stat}(L) + \Phi_B^{(1)}(L)$$

Example of results: $\Phi_B(L_1)$



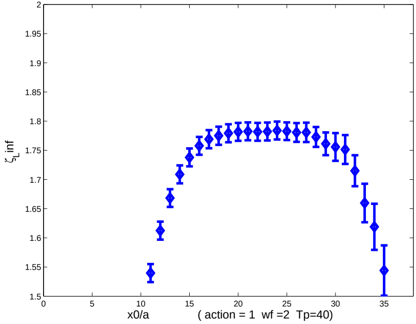
L_1/a	12	10	8	6
β	6.78	6.64	6.45	6.22

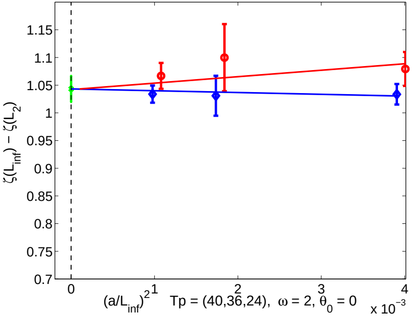
Example of results: $\Phi_B(L_2)$



L_2/a	24	20	16	12
β	6.78	6.64	6.45	6.22

Example of results in the large volume

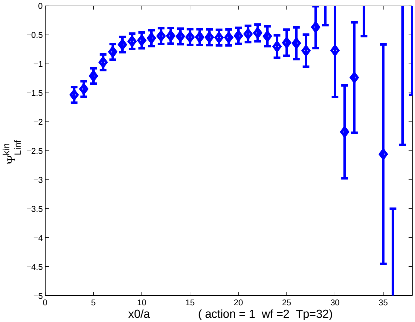


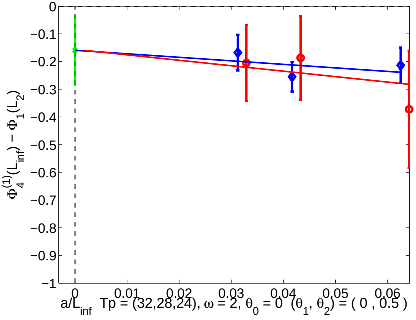


$$L_\infty = 4L_1 \simeq 1.5 \text{ fm}$$

L/a	32	24	16
β	6.45	6.3	6.0

Example of results in the large volume





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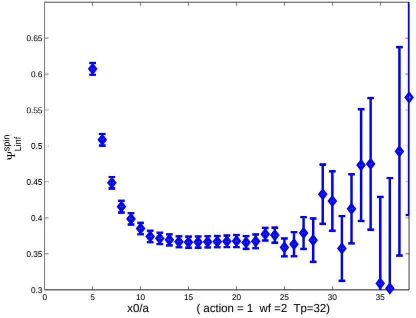
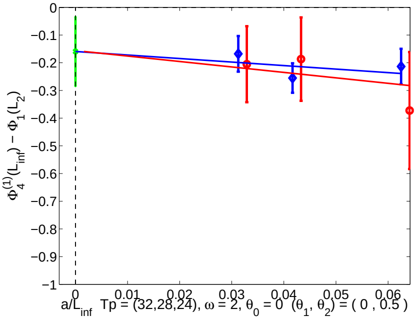




$$L_\infty = 4L_1 \simeq 1.5 \text{ fm}$$

L/a	32	24	16
β	6.45	6.3	6.0

Quenched preliminary results in MeV

Statistical errors only

Cont extr very preliminary

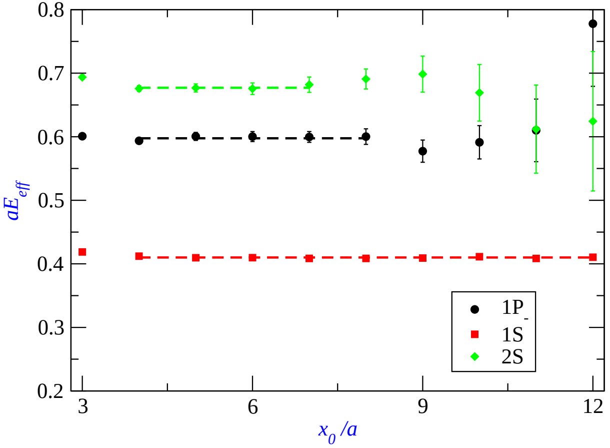
Interpolation at the b-quark mass should be done

	$F_{B_s}^{\rm stat}$	$F_{B_s}^{\rm stat} + F_{B_s}^{(1)}$		
θ_0		$\theta_1 = 0$ $\theta_2 = 0.5$	$\theta_1 = 0.5$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
0	228 ± 5	200 ± 12	201 ± 12	204 ± 12
0.5	224 ± 5	200 ± 12	202 ± 12	205 ± 12
1	212 ± 5	201 ± 12	202 ± 12	206 ± 12

Table: Results in MeV of F_{B_s}, for the mass $z = 12.1$

- The $1/m$ correction can give a $\sim 5 - 15\%$ contribution
- Adding the $1/m$ correction makes the agreement much better

Improvement: all to all propagators



Conclusion and outlook

- Computation of m_{b} and F_{B_s} Folllowing [Heitger & Sommer '03]
- Full non-perturbative calculation
- Exact cancelation of the power divergences
- Result includes NLO of HQET
- Find in the quenched approximation

$$m_{\mathrm{b}}(m_{\mathrm{b}}) = \underbrace{4.350(64)}_{\text{static}} \ \mathrm{GeV} \underbrace{-0.049(29)}_{\mathrm{O}(\Lambda^2/m_{\mathrm{b}})} \ \mathrm{GeV}$$

- and (preliminary)

$$\begin{aligned} F_{B_s}^{\mathrm{stat}} &= 228 \pm 5 \pm ?? \ \mathrm{MeV} + \mathcal{O}(1/m) \\ F_{B_s}^{\mathrm{stat}} + F_{B_s}^{(1)} &= 200 \pm 12 \pm ?? \ \mathrm{MeV} + \mathcal{O}(1/m^2) \end{aligned}$$

- Still working on improvements (extraction of the matrix elements in the large volume)
- Promising for a precise determination of F_B and F_{B_s}, using dynamical fermions (in progress).